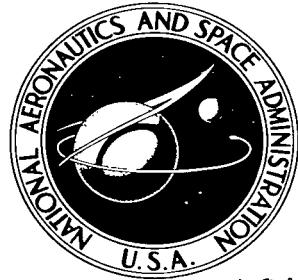


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MAGNETIC FIELD FROM A FINITE THIN CONE BY USE OF LEGENDRE POLYNOMIALS

by Lawrence Flax and Edmund E. Callaghan
Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1964



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SUMMARY

The use of zonal harmonics to evaluate magnetic fields generated by axisymmetric current sheets for all regions either near or far from the current elements is demonstrated for the particular case of a right-circular cone. Solutions of the equations for points within and outside the current sheet are presented in tables and figures.

INTRODUCTION

The advent of new superconducting materials capable of carrying large currents in strong magnetic fields has stimulated considerable interest in the use of superconducting electromagnets for a wide variety of applications, for example, plasma containment, shielding of spacecraft from high-energy-charged particles, and lossless transformers.

Calculations of the fields generated by various axisymmetric electromagnet configurations, such as loops and solenoids, have been accomplished either in terms of tabulated functions (elliptic integrals, ref. 1) or by numerical integration procedures (ref. 2). Accurate numerical evaluations of elliptic integrals are limited to certain ranges of the modulus, and as yet, expressions for the magnetic fields for certain coil configurations have not been put into complete elliptic form.

Zonal harmonics, that is, ordinary or associated Legendre polynomials, can also be used to obtain the fields of axisymmetric electrostatic and magnetostatic configurations (ref. 3). Their use for magnetic field calculations, however, has been restricted to cases that involve rapid convergence of Legendre polynomials, namely, for field points far from any current element in the source structure. In the work presented herein this restriction is removed.

It should be noted that there is a characteristic difference in the use of zonal harmonics for electrostatic or magnetostatic cases. Electrostatic fields involve the use of ordinary Legendre polynomials, whereas magnetic fields re-



quire the use of both ordinary and associated Legendre functions. This can be attributed to the fact that, in the electrostatic case, a scalar potential is used, while in the magnetostatic case, a vector potential is used.

The aim of this paper is to demonstrate the use of zonal harmonics in evaluating magnetic field components. Solutions in terms of Legendre polynomials are obtained for all points including the current elements.

SYMBOLS

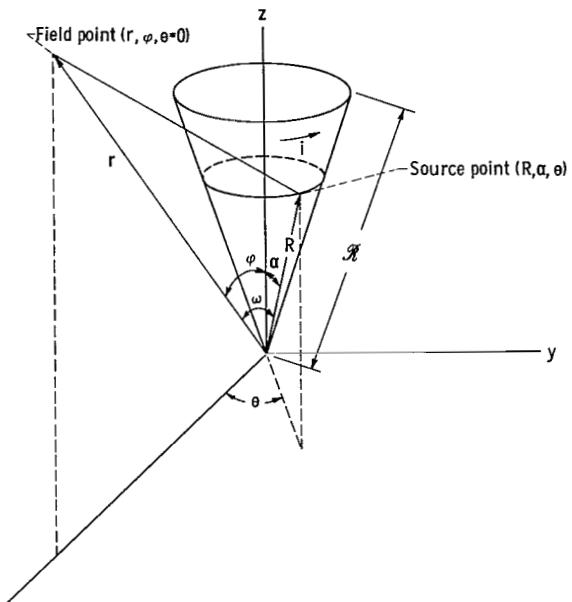
$A_\theta(r, \phi)$	magnetic vector potential
B_r, B_ϕ	components of magnetic induction in r - and ϕ -directions, respectively
i	current
N	number of turns per unit length of coil surface
P_n	Legendre polynomial of first kind
P_n^m	associated Legendre polynomial
R	distance of current element from origin
R, α, θ	spherical coordinates of source point (see sketch)
R	lateral length of cone
$r, \phi, \theta=0$	spherical coordinates of field point (see sketch)
α	conical half-angle
μ	permeability
$\cos \omega$	$\cos \alpha \cos \phi + \sin \alpha \sin \phi \cos \theta$

DERIVATION OF EQUATIONS

Consider a conical current sheet in spherical coordinates as shown in sketch (a).

For a circular current filament, the vector potential has only one component, namely, A_θ parallel to the filament, and is given by:

$$A_\theta = \frac{\mu i}{4\pi} \int_0^{2\pi} \frac{ds}{x}$$



(a)

where ds is an element of the current path and x is the distance from the element to the field point. In spherical coordinates, A_θ for a single loop (not centered at the origin) is given by

$$A_\theta = \frac{\mu i}{4\pi} \int_0^{2\pi} \frac{R \sin \alpha \cos \theta d\theta}{\sqrt{r^2 + R^2 - 2rR \cos \omega}} \quad (1a)$$

where

$$\cos \omega = \cos \alpha \cos \phi + \sin \alpha \sin \phi \cos \theta$$

and the field point is assumed to lie in the x - z plane ($\theta = 0$).

For a conical current sheet consisting of N filaments per unit of surface length and with the same current in each filament,

$$A_\theta = \frac{\mu N i}{2\pi} \int_0^\pi \int_0^R \frac{R \sin \alpha \cos \theta d\theta dR}{\sqrt{r^2 + R^2 - 2rR \cos \omega}} \quad (1b)$$

The reciprocal of the distance from the current element to the field point $1/\sqrt{r^2 + R^2 - 2rR \cos \omega}$ can be written in terms of a Legendre expansion as follows:

$$\frac{1}{r} \left(1 - \frac{2R}{r} \cos \omega + \frac{R^2}{r^2} \right)^{-1/2} = \frac{1}{r} \sum_{n=0}^{\infty} \left(\frac{R}{r} \right)^n P_n(\cos \omega) \quad \text{for } r > R$$

$$\frac{1}{R} \left(1 - \frac{2r}{R} \cos \omega + \frac{r^2}{R^2} \right)^{-1/2} = \frac{1}{R} \sum_{n=0}^{\infty} \left(\frac{r}{R} \right)^n P_n(\cos \omega) \quad \text{for } R > r$$

The first series is convergent when $R < r$, while the second series is convergent when $r < R$. Both series diverge at $r = R$ unless they can be terminated.

Magnetic Field Components in r - and φ -Directions for $r > R$

For magnetic field calculations that involve $r > R$, the vector potential can be put into Legendre form immediately:

$$\begin{aligned} A_\theta &= \frac{\mu Ni}{2\pi} \int_0^\pi \int_0^R \sin \alpha \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^{n+1} \cos \theta P_n(\cos \omega) dR d\theta \\ &= \frac{\mu Ni}{2\pi} \int_0^\pi \sin \alpha \sum_{n=0}^{\infty} \frac{P_n(\cos \omega)}{n+2} \frac{R^{n+2}}{r^{n+1}} \cos \theta d\theta \end{aligned} \quad (2a)$$

Using the addition theorem for Legendre polynomials, namely,

$$P_n(\cos \omega) = P_n(\cos \alpha)P_n(\cos \varphi) + 2 \sum_{m=1}^n \frac{(n-m)!}{(n+m)!} P_n^m(\cos \alpha)P_n^m(\cos \varphi) \cos m\theta \quad (2b)$$

yields

$$A_\theta = \frac{\mu Ni \sin \alpha}{2} \sum_{n=1}^{\infty} \frac{R^{n+2}}{r^{n+1}} \frac{1}{n(n+1)} \frac{1}{n+2} P_n^1(\cos \alpha)P_n^1(\cos \varphi) \quad (2c)$$

The term involving $n = 0$ vanishes, and use is made of the following integrals:

$$\int_0^\pi \cos \theta \cos m\theta d\theta = \begin{cases} 0 & m \neq 1 \\ \frac{\pi}{2} & m = 1 \end{cases}$$

The magnetic field B can be expressed as the curl of the vector potential A , that is,

$$\bar{B} = \nabla \times \bar{A}$$

so that the magnetic field components are given by

$$B_r = \frac{1}{r \sin \varphi} \frac{\partial (A_\theta \sin \varphi)}{\partial \varphi}$$

$$-B_\varphi = \frac{1}{r} \frac{\partial (r A_\theta)}{\partial r}$$

$$B_\theta = 0$$

Using the equation relating associated and ordinary Legendre polynomials, that is,

$$P_n^m(\cos \varphi) = (\sin^2 \varphi)^{m/2} \frac{d^m}{d(\cos \varphi)^m} P_n(\cos \varphi) \quad (2d)$$

and the Legendre differential equation

$$-\frac{1}{\sin \varphi} \frac{d}{d\varphi} \left[\sin \varphi \frac{dP_n(\cos \varphi)}{d\varphi} \right] = n(n+1)P_n(\cos \varphi) \quad (2e)$$

yields

$$B_r = \frac{\mu N_i \sin \alpha}{2} \sum_{n=1}^{\infty} \left(\frac{R}{r} \right)^{n+2} \frac{1}{n+2} P_n^1(\cos \alpha) P_n(\cos \varphi) \quad (2f)$$

The B_φ field is given by

$$-B_\varphi = \frac{\mu N_i \sin \alpha}{2} \sum_{n=1}^{\infty} \left(\frac{R}{r} \right)^{n+2} \frac{P_n^1(\cos \alpha) P_n^1(\cos \varphi)}{(n+1)(n+2)} \quad (2g)$$

Magnetic Field Component in φ -Direction for $R > r$

For $r > R$, no difficulty of convergence of the series was encountered. In the present case, however, the integration over R passes through the value $r = R$, and the series diverges. This is true regardless of the value of the conical half-angle α .

In order to avoid this difficulty, it is necessary to integrate equation (1b) with respect to R , perform the curl operation to obtain the B_φ component, and then manipulate the terms into Legendre form. The integration with respect to R yields

$$\begin{aligned}
 \frac{2\pi A_\theta}{\mu N_i \sin \alpha} = & \\
 + \int_0^\pi \frac{\cos \theta d\theta}{Q} & \quad I(a) \\
 + \int_0^\pi r \cos \omega \cos \theta \ln \left(R - r \cos \omega + \frac{1}{Q} \right) d\theta & \quad II(a) \\
 - \int_0^\pi r \cos \omega \cos \theta \ln(r - r \cos \omega) d\theta & \quad III(a)
 \end{aligned}
 \quad \left. \right\} (3a)$$

Performing the curl operation then gives

$$\begin{aligned}
 \frac{2\pi B_\varphi}{\mu N_i \sin \alpha} = & \\
 - \int_0^\pi Q R \sin \alpha \sin \varphi d\theta & \quad I(b) \\
 + \int_0^\pi Q \cos \theta (r - R \cos \alpha \cos \varphi) d\theta & \quad II(b) \\
 - \int_0^\pi Q R \cos \theta \cos \omega d\theta & \quad III(b) \\
 + 2 \int_0^\pi \cos \omega \cos \theta \ln \left(R - r \cos \omega + \frac{1}{Q} \right) d\theta & \quad IV \\
 - 2 \int_0^\pi \cos \omega \cos \theta \ln[r(1 - \cos \omega)] d\theta & \quad V
 \end{aligned}
 \quad \left. \right\} (3b)$$

where

$$Q = \frac{1}{\sqrt{R^2 + r^2 - 2Rr \cos \omega}}$$

After equation (3b) is integrated with respect to θ , the first three terms can be put into appropriate Legendre form:

$$\begin{aligned}
 I(b) + II(b) + III(b) &= -\frac{3}{2} \pi \sum_{n=0}^{\infty} \left(\frac{r}{R}\right)^n \sin \alpha \sin \varphi P_n(\cos \alpha) P_n(\cos \varphi) \\
 &\quad + \pi \sum_{n=1}^{\infty} \left(\frac{r}{R} - 2 \cos \alpha \cos \varphi\right) \left(\frac{r}{R}\right)^n \frac{P_n^1(\cos \alpha) P_n^1(\cos \varphi)}{n(n+1)} \\
 &\quad - \frac{\pi}{2} \sum_{n=2}^{\infty} \left(\frac{r}{R}\right)^n \sin \alpha \sin \varphi \frac{(n-2)!}{(n+2)!} P_n^2(\cos \alpha) P_n^2(\cos \varphi)
 \end{aligned}$$

Term IV of equation (3b) can now be put into suitable form by differentiating with respect to r , where r is not a variable of integration. After considerable algebraic manipulation, the fourth term of equation (3b) can be written

$$IV = -2 \int_0^\pi \frac{QR \cos \omega \cos \theta}{r} d\theta + 2 \int_0^\pi \frac{\cos \omega \cos \theta}{r} d\theta$$

Term IV can now be put into suitable zonal harmonic form. Then, integrating with respect to r to negate the differentiation and evaluating the constant of integration by suitable boundary conditions reduce term IV to

$$\begin{aligned}
 IV &= -2\pi \cos \alpha \cos \varphi \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^n \frac{1}{n^2(n+1)} P_n^1(\cos \alpha) P_n^1(\cos \varphi) \\
 &\quad - \pi \sin \alpha \sin \varphi \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^n \frac{1}{n} P_n(\cos \alpha) P_n(\cos \varphi) \\
 &\quad - \pi \sin \alpha \sin \varphi \sum_{n=2}^{\infty} \frac{1}{n} \frac{(n-2)!}{(n+2)!} \left(\frac{r}{R}\right)^n P_n^2(\cos \alpha) P_n^2(\cos \varphi) \\
 &\quad + \pi \sin \alpha \sin \varphi \ln 2R
 \end{aligned}$$

Term V can be integrated without resorting to Legendre functions and is equal to

$$V = \sin \alpha \sin \varphi \left[\ln \left| \left(1 + \sqrt{1 - z^2} \right) (1 - \cos \alpha \cos \varphi) \right| - \left(\frac{1 - \sqrt{1 - z^2}}{z^2} \right) \right. \\ \left. + \frac{1}{2} - \ln 2 + \ln r \right] - 2 \left(\frac{1 - \sqrt{1 - z^2}}{z} \right) \cos \alpha \cos \varphi$$

where

$$z = \frac{-\sin \alpha \sin \varphi}{1 - \cos \alpha \cos \varphi}$$

The final expression for B_φ for $\mathfrak{R} > r$ becomes:

$$\frac{2\pi B_\varphi}{\mu Ni} = \pi \sin \alpha \left\{ \sum_{n=1}^{\infty} \left(\frac{r}{\mathfrak{R}} \right)^n \frac{P_n^1(\cos \alpha) P_n^1(\cos \varphi)}{n(n+1)} \left[\frac{r}{\mathfrak{R}} - 2 \left(1 + \frac{1}{n} \right) \cos \alpha \cos \varphi \right] \right. \\ \left. - \sin \alpha \sin \varphi \sum_{n=1}^{\infty} \left(\frac{r}{\mathfrak{R}} \right)^n \left(\frac{3}{2} + \frac{1}{n} \right) P_n(\cos \alpha) P_n(\cos \varphi) \right. \\ \left. - \left(\frac{3}{2} - \ln 2 \mathfrak{R} \right) \sin \alpha \sin \varphi - \sin \alpha \sin \varphi \sum_{n=2}^{\infty} \left(\frac{r}{\mathfrak{R}} \right)^n \frac{P_n^2(\cos \alpha) P_n^2(\cos \varphi)}{2(n+1)n^2(n-1)} \right. \\ \left. - \sin \alpha \sin \varphi \left[\ln \left| 1 + \sqrt{1 - z^2} (1 - \cos \alpha \cos \varphi) \right| - \left(\frac{1 - \sqrt{1 - z^2}}{z^2} \right) \right. \right. \\ \left. \left. + \frac{1}{2} - \ln 2 + \ln r \right] - 2 \left(\frac{1 - \sqrt{1 - z^2}}{z} \right) \cos \alpha \cos \varphi \right\} \quad (3c)$$

Magnetic Field Component in r -Direction for $\mathfrak{R} > r$

The B_r component for $\mathfrak{R} > r$ is obtained by first integrating the vector potential (eq. (1b)) with respect to R , which results in equation (3a). The following mathematical operations were then performed on equation (3a): In

dealing with term I(a), it is convenient first to take the curl and then apply the Legendre expansion. Terms II(a) and III(a) are closely analogous to terms IV and V, respectively, of equation (3b). Note, however, that terms II(a) and III(a) are still in vector potential form, whereas terms IV and V have already had the curl operation performed. In general, the same techniques were applied to terms II(a) and III(a) prior to the curl operation as had been used on terms IV and V after the curl operation in order to achieve similar forms. Subsequently, in applying the curl operation to II(a), the following recursion formulas were used:

$$(\cos^2 \varphi - 1) \frac{dP_n^m(\cos \varphi)}{d \cos \varphi} - (n - m + 1) P_{n+1}^m(\cos \varphi) + (n + 1) \cos \varphi P_n^m(\cos \varphi) = 0$$

or a slightly altered form

$$\frac{d}{d\varphi} P_n^m(\cos \varphi) = \frac{(n - m + 1) P_{n+1}^m(\cos \varphi) - (n - 1) \cos \varphi P_n^m(\cos \varphi)}{\sin \varphi}$$

The term III(a) can be integrated without resort to Legendre functions, as was done previously with term V. The resulting expression for B_r is

$$\begin{aligned} \frac{2\pi B_r}{\mu N_i} &= \pi \sin \alpha \left(\sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^n \frac{\cos \alpha P_n^1(\cos \alpha)}{n} \left[\frac{\sin \varphi P_n^1(\cos \varphi)}{n} - \cos \varphi P_n(\cos \varphi) \right] + \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^n \frac{\sin \alpha P_n(\cos \alpha)}{n} \right. \\ &\quad \times \left. \left[\frac{\sin \varphi P_n^1(\cos \varphi)}{2} - \cos \varphi P_n(\cos \varphi) \right] - \sum_{n=0}^{\infty} \left(\frac{r}{R}\right)^n \sin \alpha \cos \varphi [P_n(\cos \alpha) P_n(\cos \varphi)] \right. \\ &\quad - \sum_{n=2}^{\infty} \left(\frac{r}{R}\right)^n \frac{\sin \alpha P_n^2(\cos \alpha)}{2n^2(n+2)(n+1)} \left[P_{n+1}^2(\cos \varphi) - \cos \varphi P_n^2(\cos \varphi) \right] - \sin \alpha \cos \varphi \ln\left(\frac{r}{2R}\right) - \frac{\cos \alpha}{\sin \varphi} \\ &\quad \times \left[\cos \varphi \left(\frac{\cos \varphi - \cos \alpha}{1 - \cos \alpha \cos \varphi} \right) \left(\frac{1 - \sqrt{1 - z^2}}{z} \right) + \left(\frac{1 - \sqrt{1 - z^2}}{z} \right) (\cos^2 \varphi - \sin^2 \varphi) \right] \\ &\quad + \frac{z}{2} \left\{ \cos \alpha \sin \varphi + \frac{\sin \alpha (\cos \varphi - \cos \alpha)}{1 - \cos \alpha \cos \varphi} \left[\frac{z}{(1 + \sqrt{1 - z^2}) \sqrt{1 - z^2}} - \frac{z^2 - 2 + 2\sqrt{1 - z^2}}{z^3 \sqrt{1 - z^2}} \right] \right. \\ &\quad \left. - \sin \alpha \cos \varphi \left[\ln \left| \frac{(1 - \cos \alpha \cos \varphi)(1 - \sqrt{1 - z^2})}{2} \right| + \frac{\sqrt{1 - z^2} - 1}{z^2} + \frac{1}{z} \right] \right\} \end{aligned} \quad (4)$$

where

$$z = - \frac{\sin \alpha \sin \varphi}{1 - \cos \alpha \cos \varphi}$$

RESULTS AND DISCUSSION

The equations for the B_r and B_ϕ components (eqs. (2f), (2g), (3c), and (4)) were programmed for computation on a computer. For field points outside the cone and for values of r/R or R/r not near unity, the polynomials converged to an unchanging value (seven significant figures) using 30 terms or less. To achieve the same result for values of r/R or R/r near unity, small cone angles, and angles φ nearly equal to the cone angles required several hundred terms.

To check the accuracy of the method in this range of values, results were compared with those obtained by Simpson and Gaussian numerical integrations of the curl of the vector potential. For the Simpson integration, sufficiently small increments were used so that six consistent significant figures were obtained, and further decreases in increment size had no effect on the sixth significant figure.

The results of calculations for a value of the conical half-angle α equal to 5° and for values of the radius ratio near unity are given in tables I to IV. Table I gives the B_ϕ component for $R/r = 0.95, 0.96, 0.97$, and 0.99 . Table II gives B_ϕ for inverse values of r/R . Table III shows the B_r component for R/r values of $0.95, 0.96, 0.97$, and 0.99 . Table IV shows the B_r component for inverse values of r/R .

Comparison of the values obtained with the Legendre expansion and the Simpson method show that, in general, the absolute deviation is small except when r/R or $R/r = 0.99$. Even for these cases the percentage deviation is not large when the absolute values are significant. The percentage deviation can be quite large, however, for those cases where the absolute values of the components are very small, that is, of the order of 10^{-4} .

The B_ϕ and B_r values for various conical half-angles are shown in figures 1 and 2, respectively. In these figures the dimensionless field $2\pi B/\mu Ni$ is plotted as a function of field angle φ for various radius ratios r/R . As might be expected, the B_ϕ component is zero on the axis of symmetry, increases to a maximum where the angle $\varphi = \alpha$, and then decreases outside the cone. The B_r field is a maximum on the axis of symmetry for positions near the apex (small values of r/R), and the field is discontinuous at the cone surface. It should be noted that the B_ϕ field is infinite at $r/R = 1.0$ and $\alpha = \varphi$, as would be expected at the edge of a current sheet.

CONCLUDING REMARKS

The method of zonal harmonics is a powerful tool for the calculation of magnetic fields when axisymmetry of the current elements exists. With the methods contained herein, it appears that almost any reasonable configuration with zero or finite thickness can be handled. The final forms of the equa-

tions, since they can be directly differentiated, should be extremely useful in the calculation of charged particle trajectories.

Lewis Research Center

National Aeronautics and Space Administration
Cleveland, Ohio, April 7, 1964

REFERENCES

1. Callaghan, Edmund E., and Maslen, Stephen H.: The Magnetic Field of a Finite Solenoid. NASA TN D-465, 1960.
2. Brown, Gerald V., Flax, Lawrence, Itean, Eugene C., and Laurence, James C.: Axial and Radial Magnetic Fields of Thick, Finite-Length Solenoids. NASA TR R-170, 1963.
3. Garrett, Milan W.: Axially Symmetric Systems for Generating and Measuring Magnetic Fields, Pt. I. J. Appl. Phys., vol. 22, 1951, pp. 1091-1107.

TABLE I. - VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO[Conical half-angle, 5° .](a) $R/r = 0.95$

Field angle, ϕ , deg	Values of B_ϕ obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
	Deviation	Percent deviation	Deviation		Deviation	Percent deviation	Deviation	Percent deviation
1	-0.16357955	-0.16357932	-0.16358095	225	+0.00000140	-0.0009	+0.00000163	-0.0010
2	-.32337714	-.32337674	-.32337815	227	.00000101	-.0003	.00000141	-.0004
3	-.46920913	-.46920796	-.46921375	227	.00000462	-.0010	.00000579	-.0012
4	-.57756758	-.57756326	-.57757156	225	.00000398	-.0007	.00000830	-.0014
5	-.61598386	-.61581851	-.61598906	222	.00000520	-.0008	.00017055	-.0277
6	-.57637793	-.57637748	-.57638246	226	.00000453	-.0008	.00000498	-.0009
8	-.40418994	-.40418988	-.40419216	225	.00000222	-.0005	.00000228	-.0006
10	-.26588784	-.26588771	-.26588851	229	.00000067	-.0003	.00000080	-.0003
12	-.17982068	-.17981489	-.17982105	264	.00000037	-.0002	.00000616	-.0034
14	-.12637521	-.12637517	-.12637537	297	.00000016	-.0001	.00000020	-.0002
16	-.091890440	-.091890413	-.091890544	294	.00000010	-.0001	.00000013	-.0001
18	-.068724483	-.068724440	-.068724547	291	.00000006	-.0001	.00000013	-.0002
20	-.052600771	-.052600783	-.052600805	290	.00000003	-.0001	.00000002	-.0000
25	-.029112426	-.029112414	-.029112440	262	.00000001	-.0000	.00000003	-.0001
30	-.017433017	-.017433012	-.017433028	291	.00000001	-.0001	.00000002	-.0001
35	-.011016916	-.011016912	-.011016922	291	.00000001	-.0001	.00000001	-.0001
40	$-7.2331364 \times 10^{-3}$	$-7.2331236 \times 10^{-3}$	$-7.2331415 \times 10^{-3}$	293	.00000001	-.0001	.00000002	-.0002
45	$-4.8810476 \times 10^{-3}$	$-4.8810392 \times 10^{-3}$	$-4.8810539 \times 10^{-3}$	294	.00000001	-.0001	.00000001	-.0003
50	$-3.3587499 \times 10^{-3}$	$-3.3587360 \times 10^{-3}$	$-3.3587597 \times 10^{-3}$	297	.00000001	-.0003	.00000002	-.0007
55	$-2.3419248 \times 10^{-3}$	$-2.3419070 \times 10^{-3}$	$-2.3419321 \times 10^{-3}$	297	.00000001	-.0003	.00000003	-.0011
60	$-1.6455397 \times 10^{-3}$	$-1.6454986 \times 10^{-3}$	$-1.6455430 \times 10^{-3}$	297	.00000000	-.0002	.00000004	-.0027
70	$-8.1377933 \times 10^{-4}$	$-8.1376508 \times 10^{-4}$	$-8.1378127 \times 10^{-4}$	299	.00000000	-.0002	.00000002	-.0020
80	$-3.8606403 \times 10^{-4}$	$-3.8605981 \times 10^{-4}$	$-3.8606653 \times 10^{-4}$	301	.00000000	-.0006	.00000001	-.0017
90	$-1.5939581 \times 10^{-4}$	$-1.5937016 \times 10^{-4}$	$-1.5939844 \times 10^{-4}$	301	.00000000	-.0016	.00000003	-.0177
100	$-3.8936997 \times 10^{-5}$	$-3.8890466 \times 10^{-5}$	$-3.8938350 \times 10^{-5}$	301	.00000000	-.0035	.00000005	-.1231
120	5.1177267×10^{-5}	$+5.1148046 \times 10^{-5}$	$+5.1176489 \times 10^{-5}$	301	.00000000	+.0015	-.00000003	-.0556
140	5.7030017×10^{-5}	5.7179634×10^{-5}	5.7027651×10^{-5}	301	.00000000	.0041	.00000015	.2658
160	3.3306531×10^{-5}	3.3181204×10^{-5}	3.3318833×10^{-5}	301	-.00000001	-.0369	-.00000014	-.4148

TABLE I. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO
 [Conical half-angle, 5° .]
 (b) $R/r = 0.96$

Field angle, ϕ , deg	Values of B_ϕ Gauss	obtained by method of -			Number of terms	Values obtained by method of -			
		Simpson	Legendre			Gauss		Simpson	
						Deviation	Percent deviation	Deviation	Percent deviation
1	-0.19339033	-0.19338956	-0.19339322	300	+0.00000289	-0.0015	+0.00000366	-0.0019	
2	-.38814861	-.38814820	-.38815416	287	+.00000555	-.0014	+.00000596	-.0015	
3	-.57756928	-.57756768	-.57757991	261	+.00001063	-.0018	.00001223	-.0021	
4	-.73097655	-.73097141	-.73098791	263	.00001136	-.0016	.00001650	-.0023	
5	-.78455670	-.78413118	-.78456951	262	.00001281	-.0016	.00043833	-.0559	
6	-.71106972	-.71107070	-.71108203	262	.00001231	-.0017	.00001133	-.0016	
8	-.45664964	-.45664874	-.45665459	265	.00000495	-.0011	.00000585	-.0013	
10	-.28589020	-.28588986	-.28589152	301	.00000131	-.0005	.00000166	-.0006	
12	-.18853077	-.18853058	-.18853138		.00000061	-.0003	.00000080	-.0004	
14	-.13060853	-.13060840	-.13060876		.00000023	-.0002	.00000036	-.0003	
16	-.094112260	-.094112229	-.094112458		.00000020	-.0002	.00000023	-.0002	
18	-.069951200	-.069951121	-.069951271		.00000007	-.0001	.00000015	-.0002	
20	-.053298034	-.053298028	-.053298104		.00000007	-.0001	.00000008	-.0001	
25	-.029278577	-.029278557	-.029278599		.00000002	-.0001	.00000004	-.0001	
30	-.017446965	-.017446959	-.017446937		-.00000003	+.0002	-.00000002	+.0001	
35	-.010983523	-.010983517	-.010983526		+.00000000	-.0000	+.00000001	-.0001	
40	$-7.1864520 \times 10^{-3}$	$-7.1864365 \times 10^{-3}$	$-7.1864575 \times 10^{-3}$		+.00000001	-.0001	.00000002	-.0003	
45	$-4.8330529 \times 10^{-3}$	$-4.8330410 \times 10^{-3}$	$-4.8330617 \times 10^{-3}$.00000001	-.0002	.00000002	-.0004	
50	$-3.3136714 \times 10^{-3}$	$-3.3136561 \times 10^{-3}$	$-3.3136835 \times 10^{-3}$.00000001	-.0004	.00000003	-.0008	
55	$-2.3010614 \times 10^{-3}$	$-2.3010434 \times 10^{-3}$	$-2.3010690 \times 10^{-3}$.00000001	-.0003	.00000003	-.0011	
60	$-1.6090530 \times 10^{-3}$	$-1.6090114 \times 10^{-3}$	$-1.6090546 \times 10^{-3}$.00000000	-.0001	.00000004	-.0027	
70	$-7.8510197 \times 10^{-4}$	$-7.8508768 \times 10^{-4}$	$-7.8510386 \times 10^{-4}$.00000000	-.0002	.00000002	-.0021	
80	$-3.6353661 \times 10^{-4}$	$-3.6353275 \times 10^{-4}$	$-3.6354113 \times 10^{-4}$.00000000	-.0012	.00000001	-.0023	
90	$-1.4160738 \times 10^{-4}$	$-1.4158032 \times 10^{-4}$	$-1.4160809 \times 10^{-4}$.00000000	-.0005	.00000003	-.0196	
100	$-2.4828698 \times 10^{-5}$	$-2.4782127 \times 10^{-5}$	$-2.4829854 \times 10^{-5}$.00000000	-.0047	.00000005	-.1926	
120	6.0014967×10^{-5}	5.9985035×10^{-5}	6.0015423×10^{-5}		-.00000000	-.0008	-.00000003	-.0507	
140	6.2243035×10^{-5}	6.2392703×10^{-5}	6.2239880×10^{-5}		.00000000	.0051	.00000015	.2449	
160	3.5737525×10^{-5}	3.5608677×10^{-5}	3.5747818×10^{-5}		-.00000001	-.0288	-.00000014	-.3908	

TABLE I. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO[Conical half-angle, 5° .](c) $R/r = 0.97$

Field angle, ϕ , deg	Values of B_ϕ obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
	Deviation	Percent deviation	Deviation		Deviation	Percent deviation	Deviation	Percent deviation
1	-0.22272618	-0.22272602	-0.22274583	301	+0.00001965	-0.0088	-0.00001981	-0.0089
2	-.45507039	-.45506999	-.45508242		+.00001203	-.0026	.00001243	-.0027
3	-.70053445	-.70053176	-.70054025		.00000580	-.0008	.00000849	-.0012
4	-.93016030	-.93015782	-.93016578		.00000548	-.0006	.00000796	-.0009
5	-1.0208761	-1.0204758	-1.0208929		.00001680	-.0016	.00041710	-.0409
6	-.88239446	-.88239192	-.88240529		.00001083	-.0012	.00001337	-.0015
8	-.50762669	-.50762573	-.50763385		.00000716	-.0014	.00000812	-.0016
10	-.30304590	-.30304563	-.30304769		.00000179	-.0006	.00000206	-.0007
12	-.19553155	-.19553163	-.19553103		-.00000052	+.0003	-.00000060	+.0003
14	-.13385283	-.13385280	-.13385358		+.00000075	-.0006	+.00000078	-.0006
16	-.095737666	-.095737636	-.095738029		.00000036	-.0004	.00000039	-.0004
18	-.070800236	-.070800223	-.070799821		-.00000042	+.0006	-.00000040	+.0006
20	-.053745441	-.053745437	-.053745748		.00000031	-.0006	.00000031	-.0006
25	-.029341080	-.029341066	-.029341042		-.00000004	+.0001	-.00000002	+.0001
30	-.017410406	-.017410398	-.017410249		-.00000016	.0009	-.00000015	.0009
35	-.010922559	-.010922558	-.010922456		-.00000010	.0009	-.00000010	.0009
40	$-7.1233983 \times 10^{-3}$	$-7.1233855 \times 10^{-3}$	$-7.1234155 \times 10^{-3}$		+.00000002	-.0002	.00000003	-.0004
45	$-4.7746977 \times 10^{-3}$	$-4.7746869 \times 10^{-3}$	$-4.7747849 \times 10^{-3}$.00000009	-.0018	.00000010	-.0021
50	$-3.2616919 \times 10^{-3}$	$-3.2616754 \times 10^{-3}$	$-3.2617693 \times 10^{-3}$.00000008	-.0024	.00000009	-.0029
55	$-2.2554090 \times 10^{-3}$	$-2.2553907 \times 10^{-3}$	2.2554171×10^{-3}		.00000001	-.0004	.00000003	-.0012
60	$-1.5691274 \times 10^{-4}$	$-1.5690861 \times 10^{-4}$	$-1.5690790 \times 10^{-4}$		-.00000005	+.0031	-.00000001	+.0005
70	$-7.5450128 \times 10^{-4}$	$-7.5448688 \times 10^{-4}$	$-7.5449956 \times 10^{-4}$		-.00000000	+.0002	+.00000001	-.0016
80	$-3.3984025 \times 10^{-4}$	$-3.3983652 \times 10^{-4}$	$-3.3988331 \times 10^{-4}$.00000004	-.0127	.00000005	-.0138
90	$-1.2306184 \times 10^{-4}$	$-1.2303657 \times 10^{-4}$	$-1.2303346 \times 10^{-4}$		-.00000003	+.0231	-.00000000	+.0025
100	$-1.0209982 \times 10^{-5}$	$-1.0163228 \times 10^{-5}$	$-1.0205495 \times 10^{-5}$		-.00000000	+.0439	.00000004	-.4159
120	6.9108149×10^{-5}	$+6.9078197 \times 10^{-5}$	6.9132513×10^{-5}		-.00000002	-.0353	-.00000005	-.0786
140	6.7587042×10^{-5}	6.7736456×10^{-5}	6.7550263×10^{-5}		+.00000004	.0544	+.00000019	.2749
160	3.8223638×10^{-5}	3.8093781×10^{-5}	3.8243131×10^{-5}		-.00000002	-.0510	-.00000015	-.3921

TABLE I. - Concluded. VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO[Conical half-angle, 5° .](d) $R/r = 0.99$

Field angle, ϕ , deg	Values of B_ϕ obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
	Deviation	Percent deviation	Deviation	Percent deviation				
1	-0.26470082	-0.26470056	-0.27296245	301	+0.00826163	-3.1211	+0.00826189	-3.1212
2	-.55806035	-.55805973	-.56023047		.00217012	-.3889	.00217074	-.3890
3	-.92453108	-.92452984	-.92028788		-.00424320	+.4590	-.00424196	+.4588
4	-1.4576261	-1.4576446	-1.4484760		-.00915010	+.6277	-.00916860	+.6290
5	-2.0310428	-2.0309420	-2.0183670		-.01267580	+.6241	-.01257500	+.6192
6	-1.3252094	-1.3252110	-1.3190708		-.00613860	+.4632	-.00614020	+.4633
8	-.58058538	-.58058438	-.58143610		+.00085072	-.1465	.00085172	-.1467
10	-.32343978	-.32343975	-.32400227		+.00056249	-.1739	.00056252	-.1739
12	-.20291982	-.20291980	-.20232870		-.00059112	+.2913	-.00059110	+.2913
14	-.13686423	-.13686421	-.13706154		+.00019731	-.1442	.00019733	-.1442
16	-.096993129	-.096993116	-.097132068		.00013894	-.1432	.00013895	-.1433
18	-.071271912	-.071271867	-.07044670		-.00022724	+.3188	-.00022720	+.3188
20	-.053844401	-.053844417	-.053954661		.00011026	-.2048	.00011024	-.2047
25	-.029144691	.029144685	-.029109401		-.00003529	+.1211	-.00003528	+.1211
30	-.017182723	-.017182716	-.017102017		-.00008071	.4697	-.00008070	+.4696
35	-.010716897	-.010716896	-.010666995		-.00004990	.4656	-.00004990	.4656
40	-6.9478288 $\times 10^{-3}$	-6.9478191 $\times 10^{-3}$	-6.9539634 $\times 10^{-3}$		+.00000613	-.0883	+.00000614	-.0884
45	-4.6267711 $\times 10^{-3}$	-4.6267618 $\times 10^{-3}$	-4.6659695 $\times 10^{-3}$		+.00003920	-.8472	+.00003921	-.8474
50	-3.1369909 $\times 10^{-3}$	-3.1369740 $\times 10^{-3}$	-3.1684113 $\times 10^{-3}$.00003142	-1.0016	.00003143	-1.0022
55	-2.1497290 $\times 10^{-3}$	-2.1497104 $\times 10^{-3}$	-2.1493362 $\times 10^{-3}$		-.00000039	+.0183	-.00000037	+.0174
60	-1.4789666 $\times 10^{-3}$	-1.4789246 $\times 10^{-3}$	-1.4541017 $\times 10^{-3}$		-.00002486	+.16812	-.00002482	1.6784
70	-6.8754264 $\times 10^{-4}$	-6.8752803 $\times 10^{-4}$	-6.8622725 $\times 10^{-4}$		-.00000132	+.1913	-.00000130	+.1892
80	-2.8895264 $\times 10^{-4}$	-2.8894903 $\times 10^{-4}$	-3.0767803 $\times 10^{-4}$		+.00001873	-6.4804	.00001873	-6.4818
90	-8.3715725 $\times 10^{-5}$	-8.3689524 $\times 10^{-5}$	-6.8865614 $\times 10^{-5}$		-.00001485	+17.7387	-.00001482	+17.7130
100	2.0554159 $\times 10^{-5}$	+2.0600568 $\times 10^{-5}$	+2.2934977 $\times 10^{-5}$		-.00000238	-11.5831	-.00000233	-11.3318
120	8.8051060 $\times 10^{-5}$	8.8020866 $\times 10^{-5}$	1.0033832 $\times 10^{-5}$		-.00001229	-13.9547	-.00001232	-13.9938
140	7.8662988 $\times 10^{-5}$	7.8811144 $\times 10^{-5}$	6.2340625 $\times 10^{-5}$		+.00001632	+20.7497	+.00001647	+20.8987
160	4.3357652 $\times 10^{-5}$	4.3229170 $\times 10^{-5}$	4.8679004 $\times 10^{-5}$		-.00000532	-12.2732	-.00000545	-12.6068

TABLE II. - VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO[Conical half-angle, 5° .](a) $r/R = 0.95$

Field angle, ϕ , deg	Values of B_ϕ obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
		Deviation	Percent deviation		Deviation	Percent deviation	Deviation	Percent deviation
1	-0.13843206	-0.13843186	-0.13843318	235	+0.000000112	-0.001	+0.000000132	-0.001
2	-.26938098	-.26938082	-.26938208	209	+.000000110	-.0004	+.000000126	-.0005
3	-.37751578	-.37751473	-.37751756	219	+.000000178	-.0005	.000000283	-.0007
4	-.43153426	-.43153277	-.43153463	212	+.000000037	-.00009	.000000186	-.0004
5	-.39301698	-.39154770	-.39031858	211	-.00269840	+.7	-.00122912	+.3
6	-.44523801	-.44523643	-.44523810	209	+.000000009	-.00002	+.000000167	-.0004
8	-.35088218	-.35088173	-.35088251	208	.000000033	-.00009	.000000078	-.0002
10	-.23770469	-.23770456	-.23770401	206	-.000000068	+.0003	-.000000055	+.0002
12	-.16223074	-.16224268	-.16223090	206	+.000000016	-.0001	-.000001178	+.007
14	-.11425161	-.11425159	-.11425206	205	+.000000045	-.0004	+.000000047	-.0004
16	-.082981967	-.082981948	-.082982560	203	+.000000069	-.0007	.000000061	-.0007
18	-.061880816	-.061880772	-.061881372	201	+.000000056	-.0009	.000000060	-.0010
20	-.047168444	-.047168458	-.047168485	204	.000000004	-.00009	.000000003	-.00005
25	-.025745511	-.025745497	-.025745795	202	.000000028	-.0011	.000000030	-.0012
30	-.015139508	-.015139503	-.015139384	200	-.000000012	+.0008	-.000000012	+.0008
35	$-9.3549014 \times 10^{-3}$	$-9.3549018 \times 10^{-3}$	$-9.3547711 \times 10^{-3}$	201	-.000000013	+.0014	-.000000013	.0014
40	$-5.9748831 \times 10^{-3}$	$-5.9748713 \times 10^{-3}$	$-5.9749066 \times 10^{-3}$	200	+.000000002	-.0004	+.000000004	-.0006
45	$-3.8969547 \times 10^{-3}$	$-3.8969445 \times 10^{-3}$	$-3.8970132 \times 10^{-3}$	199	.000000006	-.0015	.000000007	-.0018
50	$-2.5695137 \times 10^{-3}$	$-2.5694963 \times 10^{-3}$	$-2.5694829 \times 10^{-3}$	197	-.000000003	+.0012	-.000000001	+.0005
55	$-1.6962578 \times 10^{-3}$	$-1.6962397 \times 10^{-3}$	$-1.6962423 \times 10^{-3}$	199	-.000000002	+.0009	+.000000000	-.0002
60	$-1.1087926 \times 10^{-3}$	$-1.1087302 \times 10^{-3}$	$-1.1088205 \times 10^{-3}$	172	+.000000003	-.0025	+.000000007	-.0063
70	$-4.2885825 \times 10^{-4}$	$-4.2884490 \times 10^{-4}$	$-4.2884394 \times 10^{-4}$	173	-.000000001	+.0033	-.000000000	+.0002
80	$-9.9862043 \times 10^{-5}$	$-9.9858066 \times 10^{-5}$	$-9.9841075 \times 10^{-5}$	197	-.000000002	.021	-.000000002	+.018
90	5.8694268×10^{-5}	5.8722159×10^{-5}	5.8648369×10^{-5}	194	+.000000005	+.078	+.000000007	+.126
100	1.2986466×10^{-4}	1.2991073×10^{-4}	1.2991206×10^{-4}	197	-.000000005	-.036	.000000000	+.010
120	1.5382577×10^{-4}	1.5379398×10^{-4}	1.5385525×10^{-4}	198	-.000000003	-.019	.000000006	.040
140	1.1663962×10^{-4}	1.1678934×10^{-4}	1.1664478×10^{-4}	207	-.000000001	-.004	.000000014	.124
160	6.0858167×10^{-5}	6.0728067×10^{-5}	6.0762998×10^{-5}	212	+.000000010	+.16	.000000003	.058

TABLE II. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO
 [Conical half-angle, 5° .]

(b) $r/\tilde{r} = 0.96$

Field angle, ϕ , deg	Values of B_ϕ obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
					Deviation	Percent deviation	Deviation	Percent deviation
1	-0.17508889	-0.17508883	-0.17508980	279	+0.00000091	-0.0005	+0.00000097	-0.0006
2	-.34856669	-.34856645	-.34856999	276	+.00000330	-.0009	.00000354	-.0010
3	-.50895376	-.50895329	-.50895889	273	.00000513	-.0010	.00000560	-.0011
4	-.61636596	-.61636342	-.61636616	259	.00000020	-.00003	.00000274	-.0004
5	-.59586818	-.59439760	-.59317456	270	-.00269362	+.452	-.00122304	+.206
6	-.60861797	-.60861639	-.60862026	247	+.00000229	-.0004	+.00000387	-.0006
8	-.41731884	-.41731479	-.41732159	263	.00000275	-.0007	.00000680	-.0016
10	-.26489210	-.26489194	-.26489431	243	.00000221	-.0008	.00000237	-.0009
12	-.17522233	-.17521700	.17522152	245	-.00000081	+.0005	.00000452	-.0026
14	-.12132715	-.12132713	-.12132801	246	+.00000086	-.0007	+.00000087	-.0007
16	-.087234876	-.087234858	-.087235299	243	.00000042	-.0005	.00000044	-.0005
18	-.064635956	-.064635916	-.064635468	244	-.00000049	+.0008	-.00000045	-.0007
20	-.049060305	-.049060796	-.049060796	243	+.00000049	-.0010	.00000049	-.0010
25	-.026633886	-.026633862	-.026633789	243	-.00000010	.0004	-.00000007	+.0003
30	-.015638565	-.015638562	-.015638300	240	-.00000026	+.0017	-.00000026	.0017
35	$-9.6700217 \times 10^{-3}$	$-9.6700224 \times 10^{-3}$	$-9.6699731 \times 10^{-3}$	239	-.00000005	.0005	-.00000005	.0005
40	$-6.1905514 \times 10^{-3}$	$-6.1905403 \times 10^{-3}$	$-6.1906410 \times 10^{-3}$	240	+.00000009	-.0014	+.00000010	-.0016
45	$-4.0533451 \times 10^{-3}$	$-4.0533313 \times 10^{-3}$	$-4.0532770 \times 10^{-3}$	239	-.00000007	+.0017	-.00000000	+.0001
50	$-2.6878895 \times 10^{-3}$	$-2.6878735 \times 10^{-3}$	$-2.6878360 \times 10^{-3}$	237	-.00000005	+.0020	-.00000004	+.0014
55	$-1.7888352 \times 10^{-3}$	$-1.7888178 \times 10^{-3}$	$-1.7888737 \times 10^{-3}$	238	+.00000004	-.0022	+.00000006	-.0031
60	$-1.1830611 \times 10^{-3}$	$-1.1830193 \times 10^{-3}$	$-1.1830394 \times 10^{-3}$	208	-.00000002	+.0018	+.00000002	-.0017
70	$-4.7943117 \times 10^{-4}$	$-4.7941729 \times 10^{-4}$	$-4.7945949 \times 10^{-4}$	237	+.00000003	-.0059	+.00000004	-.0088
80	$-1.3617515 \times 10^{-4}$	$-1.3617106 \times 10^{-4}$	$-1.3611002 \times 10^{-4}$	236	-.00000007	+.048	-.00000006	+.045
90	3.1689297×10^{-5}	3.1715103×10^{-5}	$+3.1613934 \times 10^{-5}$	236	+.00000008	.24	.00000010	.32
100	1.0932648×10^{-4}	1.0937185×10^{-4}	1.0933069×10^{-4}	238	-.00000000	-.0039	+.00000004	+.038
120	1.4161349×10^{-4}	1.4158259×10^{-4}	1.4152466×10^{-4}	245	+.00000009	.063	.00000006	.041
140	1.0963550×10^{-4}	1.0978371×10^{-4}	1.0967091×10^{-4}	253	-.00000004	-.032	.00000011	+.103
160	5.7641192×10^{-5}	5.7512594×10^{-5}	5.7627577×10^{-5}	258	.00000001	.023	-.00000011	.200

TABLE II. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO[Conical half-angle, 5° .](c) $r/R = 0.97$

Field angle ϕ , deg	Values of B_ϕ obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
					Deviation	Percent deviation	Deviation	Percent deviation
1	-0.21047028	-0.21047043	-0.21049027	301	+0.00001999	-0.009	0.00001984	-0.009
2	-.42854709	-.42854680	-.42855801		+.00001092	-.003	.00001121	-.003
3	-.65404248	-.65404013	-.65404434		.00000186	-.0003	.00000421	-.0006
4	-.84854095	-.84853852	-.84853878		-.00000217	+.0003	+.00000026	-.00003
5	-.87018801	-.86871532	-.86749851		-.00268950	+.309	-.00121681	+.140
6	-.80956999	-.80956779	-.80957359		+.00000360	-.0004	+.000000580	-.0007
8	-.48074214	-.48074170	-.48074905		+.00000061	-.0014	+.00000735	-.0015
10	-.288333853	-.28833844	-.28834036		.00000183	-.0006	.00000192	-.0007
12	-.18603521	-.18603519	-.18603426		-.00000095	+.0003	-.00000093	.0003
14	-.12715092	-.12715090	-.12715157		.00000065	-.0005	.00000067	-.0005
16	-.090732946	-.090732934	-.090733496		.00000056	-.0006	.00000055	-.0006
18	-.066911842	-.066911778	-.066911319		-.00000052	+.0008	-.00000046	+.0007
20	-.050633655	-.050633658	-.050633882		+.00000023	-.0004	+.00000022	-.0004
25	-.027387985	-.027387975	-.027387814		-.00000017	+.0006	-.00000016	+.0006
30	-.016070825	-.016070822	-.016070652		-.00000017	.0011	-.00000017	.0011
35	$-9.9477992 \times 10^{-3}$	$-9.9477968 \times 10^{-3}$	$-9.9477922 \times 10^{-3}$		-.00000001	+.0001	-.00000000	+.00005
40	$-6.3834643 \times 10^{-3}$	$-6.3834505 \times 10^{-3}$	$-6.3835555 \times 10^{-3}$		+.00000009	-.0014	+.00000010	-.0016
45	$-4.1949216 \times 10^{-3}$	$-4.1949102 \times 10^{-3}$	$-4.1949756 \times 10^{-3}$.00000005	-.0013	.00000007	-.0016
50	$-2.7961044 \times 10^{-3}$	$-2.7960882 \times 10^{-3}$	$-2.7962897 \times 10^{-3}$	281	.00000019	-.0066	-.00000020	-.0072
55	$-1.8741482 \times 10^{-3}$	$-1.8741302 \times 10^{-3}$	$-1.8742389 \times 10^{-3}$	281	.00000009	-.0048	.00000011	-.0058
60	$-1.2519516 \times 10^{-3}$	$-1.2519083 \times 10^{-3}$	$-1.2519322 \times 10^{-3}$	279	-.00000002	+.0015	+.00000002	-.0019
70	$-5.2680878 \times 10^{-4}$	$-5.2672595 \times 10^{-4}$	$-5.2682595 \times 10^{-4}$	281	+.00000002	-.0032	+.00000003	-.0058
80	$-1.7043170 \times 10^{-4}$	$-1.7042776 \times 10^{-4}$	$-1.7042156 \times 10^{-4}$	301	-.00000001	+.0059	-.00000001	+.0036
90	6.0873523×10^{-6}	6.1129913×10^{-6}	6.0357550×10^{-6}	301	+.00000005	.85	+.00000008	1.3
100	8.9783190×10^{-5}	8.9829904×10^{-5}	8.9850086×10^{-5}	301	-.00000007	-.075	-.00000002	-.022
120	1.2993774×10^{-4}	1.2990724×10^{-4}	1.2990462×10^{-4}	301	.00000003	.025	+.00000000	.0020
140	1.0291892×10^{-4}	1.0306867×10^{-4}	1.0280560×10^{-4}	301	.00000011	.110	.00000026	.255
160	5.4551660×10^{-5}	5.4424396×10^{-5}	5.4670152×10^{-5}	301	-.00000012	-.217	-.00000025	-.452

TABLE II. - Concluded. VALUES OF MAGNETIC FIELD COMPONENT IN ϕ -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO

[Conical half-angle, 5°.]

(d) $r/R = 0.99$

Field angle, ϕ , deg	Values of B_ϕ obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
					Deviation	Percent deviation	Deviation	Percent deviation
1	-0.26138811	-0.26138798	-0.26972403	301	+0.00833592	-3.189	+0.00833605	-3.189
2	-.55104941	-.55104972	-.55324823		.00219882	-.399	.00219851	-.399
3	-.91270972	-.91270742	-.90839304		-.00431668	+.473	-.00431438	+.473
4	-1.4371690	-1.4371673	-1.4277712		-.00939780	.654	-.00939610	.654
5	-1.9711497	-----	-1.9553149		-.01583480	.803	-----	-----
6	-1.3066852	-1.3066802	-1.3002173		-.00646790	.495	-.00646290	.495
8	-.57300888	-.57300888	-.57385166		+.00084278	-.147	+.00084278	.147
10	-.31900182	-.31900158	-.31968301		.00068119	-.214	.00068143	-.214
12	-.19996139	-.19996134	-.19930692		-.00065447	.327	-.00065442	.327
14	-.13474018	-.13474017	-.13489841		+.00015823	-.117	.00015824	-.117
16	-.095390552	-.095390556	-.095608804		+.00021825	-.229	.00021825	-.229
18	-.070018407	-.070018359	-.069761207		-.00025720	+.367	-.00025715	+.367
20	-.052836596	-.052836605	-.052903277		+.00006668	-.126	.00006667	-.126
25	-.028507823	-.028507798	-.028416363		-.00009146	+.321	-.00009144	+.321
30	-.016744347	-.016744339	-.016662244		-.00008210	+.490	-.00008210	+.490
35	-.010397236	-.010397231	-.010393475		-.00000376	+.036	-.00000376	+.036
40	-.6.7048486x10 ⁻³	-.6.7048367x10 ⁻³	-.6.7438353x10 ⁻³		+.00003899	-.581	+.00003900	-.582
45	-.4.4362098x10 ⁻³	-.4.4361984x10 ⁻³	-.4.4571541x10 ⁻³		+.00002094	-.472	+.00002096	-.472
50	-.2.9838603x10 ⁻³	-.2.9838444x10 ⁻³	-.2.9723697x10 ⁻³		-.00001149	+.385	-.00001147	+.385
55	-.2.0242732x10 ⁻³	-.2.0242565x10 ⁻³	-.2.0099685x10 ⁻³		-.00001430	+.707	-.00001429	+.706
60	-.1.3745612x10 ⁻³	-.1.3745194x10 ⁻³	-.1.3799172x10 ⁻³		+.00000536	-.390	+.00000540	-.393
70	-.6.1255485x10 ⁻⁴	-.6.1254113x10 ⁻⁴	-.6.0832983x10 ⁻⁴		-.00000422	+.690	-.00000421	+.688
80	-.2.3314210x10 ⁻⁴	-.2.3313784x10 ⁻⁴	-.2.2738697x10 ⁻⁴		-.00000576	+.2469	-.00000575	+.2467
90	-.4.1159455x10 ⁻⁵	-.4.1132796x10 ⁻⁵	-.6.3922157x10 ⁻⁵		+.00002276	-.55.304	+.00002279	-.55.404
100	+.5.3509527x10 ⁻⁵	5.3554820x10 ⁻⁵	+.8.5213358x10 ⁻⁵		-.00003170	-.59.249	-.00003166	-.59.114
120	1.0810134x10 ⁻⁴	1.0807346x10 ⁻⁴	9.4056636x10 ⁻⁴		+.00001404	+.12.992	.00001402	+.12.970
140	9.0308939x10 ⁻⁵	9.0460027x10 ⁻⁵	3.8434790x10 ⁻⁵		.00005187	57.441	.00005203	57.512
160	4.8740687x10 ⁻⁵	4.8613590x10 ⁻⁵	9.8623028x10 ⁻⁵		-.00004988	-102.342	-.00005001	-102.871

TABLE III. - VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO[Conical half-angle, 5° .](a) $R/r = 0.95$

Field angle, ϕ , deg	Values of B_r obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
					Deviation	Percent deviation	Deviation	Percent deviation
1	1.2389023	1.2389049	1.2389099	225	-0.00000760	-0.0006	-0.00000500	-0.0004
2	1.1684493	1.1684521	1.1684541	227	-.00000480	-.0004	-.00000200	-.0002
3	1.0421025	1.0421110	1.0421064	227	-.00000390	-.0004	-.00000460	-.0004
4	.85590527	.85591024	.85590987	225	-.00000460	-.0005	.00000037	.0000
5	.63064990	.63179353	.63065144	222	-.00000154	-.0002	.00114209	.1808
6	.41898483	.41899160	.41898337	226	.00000146	.0003	.00000823	.0020
8	.16072501	.16072599	.16072384	225	.00000117	.0007	.00000215	.0013
10	.059862235	.059862496	.059861930	229	.00000030	.0005	.00000057	.0009
12	.020074776	.020074950	+.020074355	264	.00000042	.0021	.00000060	.0030
14	.0030417167	.0030418401	+.0030416070	297	.00000011	.0036	.00000023	.0077
16	-.0046529852	-.0046529434	-.0046531090	294	+.00000012	-.0027	.00000017	-.0036
18	-.0081539750	-.0081539516	-.0081540605	291	.00000009	-.0010	.00000011	-.0013
20	-.0096430669	-.0096430544	-.0096431230	290	.00000006	-.0006	.00000007	-.0007
25	-.0099696666	-.0099696491	-.0099697008	262	.00000003	-.0003	.00000005	-.0005
30	-.0089155205	-.0089155163	-.0089155366	291	.00000002	-.0002	.00000002	-.0002
35	-.0077358065	-.0077358023	-.0077358173	291	.00000001	-.0001	.00000002	-.0002
40	-.0066960508	-.0066960187	-.0066960572	293	.00000001	-.0001	.00000004	-.0006
45	-.0058319762	-.0058319725	-.0058319814	294	.00000001	-.0001	.00000001	-.0002
50	-.0051247173	-.0051247168	-.0051247182	297	.00000000	-.0000	.00000000	-.0000
55	-.0045460761	-.0045460748	-.0045460759	297	-.00000000	+.0000	+.00000000	-.0000
60	-.0040702519	-.0040702586	-.0040702526	297	+.00000000	-.0000	-.000000001	+.0001
70	-.0033473462	-.0033473477	-.0033473468	297	.00000000	-.0000	-.000000000	+.0000
80	-.0028368167	-.0028368267	-.0028368182	301	.00000000	-.0001	-.000000001	+.0003
90	-.0024665703	-.0024665677	-.0024665676	301	-.00000000	+.0001	-.000000000	+.0000
100	-.0021923555	-.0021923383	-.0021923548	301	-.00000000	+.0000	+.00000002	-.0008
120	-.0018301626	-.0018302106	-.0018301644	301	+.00000000	-.0001	-.000000005	+.0025
140	-.0016237072	-.0016238857	-.0016237112	301	+.00000000	-.0002	-.000000017	+.0107
160	-.0015162698	-.0015161885	-.0015162974	301	.00000003	-.0018	.00000011	-.0072

TABLE III. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION

FOR SEVERAL VALUES OF RADIUS RATIO

[Conical half-angle, 5° .](b) $r/r = 0.96$

Field angle, φ , deg	Values of B_r obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
					Deviation	Percent deviation	Deviation	Percent deviation
1	1.4812344	1.4812460	1.4812417	300	-0.00000730	-0.0005	+0.00000430	0.0003
2	1.4064544	1.4064647	1.4064596	287	-.00000520	-.0004	+.00000510	.0004
3	1.2630649	1.2630820	1.2630708	261	-.00000590	-.0005	.00001120	.0009
4	1.0299091	1.0299438	1.0299100	263	-.00000090	-.0001	.00003380	.0033
5	.72288846	.72534600	.72289144	262	-.00000298	-.0004	.00245456	.3384
6	.43718624	.43720453	.43718954	262	-.00000330	-.0008	.00001499	.0034
8	.13774966	.13775533	.13774963	265	+.00000003	+.0000	.00000570	.0041
10	.041959659	.041960520	.041959435	301	.00000022	.0005	.00000108	.0026
12	.0083862445	+.0083866099	.0083858121		.00000043	.0051	.00000080	.0095
14	-.0047035946	-.0047034298	-.0047037444		.00000015	-.0032	.00000031	-.0067
16	-.0099890224	-.0099889313	-.0099891020		.00000008	-.0008	.00000017	-.0017
18	-.011973433	-.011973383	-.011973520		.00000009	-.0007	.00000014	-.0011
20	-.012469893	-.012469865	-.012469959		.00000007	-.0005	.00000009	-.0008
25	-.011465211	-.011465190	-.011465251		.00000004	-.0003	.00000006	-.0005
30	-.0098099840	-.0098099732	-.0098099974		.00000001	-.0001	.00000002	-.0002
35	-.0083196539	-.0083196471	-.0083196575		.00000000	-.0000	.00000001	-.0001
40	-.0071029055	-.0071028722	-.0071029058		.00000000	-.0000	.00000003	-.0005
45	-.0061302730	-.0061302681	-.0061302751		.00000000	-.0000	.00000001	-.0001
50	-.0053524948	-.0053524941	-.0053524975		.00000000	-.0001	.00000000	-.0001
55	-.0047258924	-.0047258906	-.0047258950		.00000000	-.0001	.00000000	-.0001
60	-.0042161996	-.0042162060	-.0042162015		.00000000	-.0000	-.00000000	+.0001
70	-.0034501626	-.0034501642	-.0034501604		-.00000000	+.0001	-.00000000	+.0001
80	-.0029145087	-.0029145582	-.0029145107		+.00000000	-.0001	-.00000005	+.0016
90	-.0025285234	-.0025285209	-.0025285215		-.00000000	+.0001	+.00000000	-.0000
100	-.0022439042	-.0022438872	-.0022439025		-.00000000	+.0001	+.00000002	-.0007
120	-.0018695119	-.0018695597	-.0018695145		+.00000000	-.0001	-.00000005	+.0024
140	-.0016568506	-.0016570293	-.0016568556		+.00000001	-.0003	-.00000017	+.0105
160	-.0015463956	-.0015463148	-.0015464180		+.00000002	-.0014	+.00000010	-.0067

TABLE III. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION

FOR SEVERAL VALUES OF RADIUS RATIO

[Conical half-angle, 5° .](c) $\kappa/r = 0.97$

Field angle, ϕ , deg	Values of B_r obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
	Deviation	Percent deviation	Deviation	Percent deviation				
1	1.7632137	1.7632218	1.7632270	301	-0.000001330	-0.0008	-0.00000520	-0.0003
2	1.6922172	1.6922247	1.6922373		-.000002010	-.0012	-.00001260	-.0007
3	1.5439379	1.5439453	1.5439587		-.000002080	-.0013	-.00001340	-.0009
4	1.2639552	1.2639696	1.2639648		-.000000960	-.0008	+.00000480	+.0004
5	.83112845	.83297113	.83113152		-.00000307	-.0004	+.00183961	.2208
6	.43131897	.43133225	.43131366		+.000000531	+.0012	.00001859	.0043
8	.10112483	.10111924	.10111965		.00000518	.0051	-.00000041	-.0004
10	+.019718134	+.019718721	.019719376		-.00000124	-.0063	-.00000066	-.0033
12	-.0049186338	-.0049183216	-.0049193433		+.00000071	-.0144	+.00000102	-.0208
14	-.013158880	-.013158747	-.013159718		+.00000084	-.0064	.00000097	-.0074
16	-.015679071	-.015679013	-.015678615		-.00000046	+.0029	-.00000040	+.0025
18	-.015987356	-.015987332	-.015987559		+.00000020	-.0012	.00000023	-.0014
20	-.015411891	-.015411872	-.0154112247		.00000036	-.0023	.00000038	-.0024
25	-.013000270	-.013000253	-.013000549		.00000028	-.0021	.00000030	-.0023
30	-.010721666	-.010721656	-.010721740		.00000007	-.0007	.00000008	-.0008
35	-.0089123386	-.0089123298	-.0089122619		-.00000008	+.0009	-.00000007	+.0008
40	-.0075148671	-.0075148342	-.0075147616		-.00000011	+.0014	-.00000007	+.0010
45	-.0064317978	-.0064317936	-.0064317598		-.00000004	+.0006	-.00000003	+.0005
50	-.0055824628	-.0055824633	-.0055825052		+.00000004	-.0008	.00000004	-.0008
55	-.0049072797	-.0049072794	-.0049073496		+.00000007	-.0014	.00000007	-.0014
60	-.0043633278	-.0043633338	-.0043633609		.00000003	-.0008	.00000003	-.0006
70	-.0035537185	-.0035537196	-.0035536681		-.00000005	+.0014	-.00000005	+.0014
80	-.0029927177	-.0029927246	-.0029927368		+.00000002	-.0006	.00000001	-.0004
90	-.0025908662	-.0025908643	-.0025908875		.00000002	-.0008	.00000002	-.0009
100	-.0022957660	-.0022957486	-.0022957293		-.00000004	+.0016	-.00000002	+.0008
120	-.0019090881	-.0019091358	-.0019091121		+.00000002	-.0013	-.00000002	+.0012
140	-.0016901815	-.0016903597	-.0016901968		+.00000002	-.0009	-.00000016	+.0096
160	-.0015766876	-.0015766031	-.0015766637		-.00000002	+.0015	+.00000006	-.0038

TABLE III. - Concluded. VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION
 FOR SEVERAL VALUES OF RADIUS RATIO
 [Conical half-angle, 5° .]
 (d) $\kappa/r = 0.99$

Field angle, φ , deg	Values of B_r obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
					Deviation	Percent deviation	Deviation	Percent deviation
1	2.4320450	2.4320473	2.4343530	301	-0.00230800	-0.0949	-0.00230570	-0.948
2	2.4056975	2.4056996	2.4121618		-.00646430	-.2687	-.00646220	-.2686
3	2.3363903	2.3363948	2.3418141		-.00542380	-.2321	-.00541930	-.2320
4	2.1181272	2.1181358	2.1188411		-.00071390	-.0337	-.00070503	-.0333
5	1.1213462	1.1223215	1.1203424		+.00100380	+.0895	+.00197910	+.1763
6	.19483381	.19484508	.19332196		+.00151185	+.7760	.00152312	.7817
8	-.016860554	-.016860380	-.018626936		+.00176638	-10.4764	+.00176656	-10.4776
10	-.035701089	-.035700998	-.034900468		-.00080062	+2.2426	-.00080053	2.2423
12	-.035248398	-.035248280	-.035219443		-.00002896	+.0821	-.00002884	+.0818
14	-.031644519	-.031644472	-.031995969		+.00035145	-1.1106	+.00035150	-1.1108
16	-.027833263	-.027833246	-.027571241		-.00026202	+.9414	-.00026200	+.9413
18	-.024438801	-.024438774	-.024458558		+.00001976	-.0808	+.00001978	-.0810
20	-.021547289	-.021547296	-.021692745		+.00014546	-.6751	+.00014545	-.6750
25	-.016158235	-.016158219	-.016270793		+.00011256	-.6966	+.00011257	-.6967
30	-.012584341	-.012584333	-.012609138		+.00002480	-.1970	+.00002480	-.1971
35	-.010118458	-.010118451	-.010077225		-.00004123	+.4075	-.00004123	+.4074
40	-.0083511195	-.0083510857	-.0082992390		-.00005188	+.6212	-.00005185	+.6208
45	-.0070428459	-.0070428409	-.0070239624		-.00001888	+.2681	-.00001888	+.2681
50	-.0060479510	-.0060479515	-.0060678753		+.00001992	-.3294	+.00001992	-.3294
55	-.0052741214	-.0052741209	-.0053067008		.00003258	-.6177	.00003258	-.6177
60	-.0046606920	-.0046606979	-.0046755398		+.00001485	-.3186	.00001484	-.3184
70	-.0037628387	-.0037628389	-.0037389416		-.00002390	+.6351	-.00002390	+.6351
80	-.0031505678	-.0031505775	-.0031592782		+.00000871	-.2765	.00000870	-.2762
90	-.0027166497	-.0027166468	-.0027277157		.00001107	-.4073	.00001107	-.4074
100	-.0024003769	-.0024003597	-.0023833678		-.00001701	+.7086	-.00001699	+.7079
120	-.0019888962	-.0019889443	-.0019992841		+.00001039	-.5223	+.00001034	-.5199
140	-.0017573858	-.0017575638	-.0017627269		.00000534	-.3039	.00000516	-.2938
160	-.0016377561	-.0016376745	-.0016159480		-.00002181	+1.3316	-.00002173	+.3267

TABLE IV. - VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION FOR SEVERAL VALUES OF RADIUS RATIO[Conical half-angle, 5° .](a) $r/R = 0.95$

Field angle, ϕ , deg	Values of B_r obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
					Deviation	Percent deviation	Deviation	Percent deviation
1	4.4735091	4.4735142	4.4735488	227	-0.00003970	-0.0009	-0.00003460	-0.0008
2	4.5698504	4.5698525	4.5698964	222	-0.00004600	-0.0010	-0.00004390	-0.0010
3	4.7401411	4.7401444	4.4702235	212	-0.00008240	-0.0017	-0.00007910	-0.0017
4	4.9872319	4.9872354	4.9874367	209	-0.00020480	-0.0041	-0.00020130	-0.0040
5	2.1275601	2.1389205	2.1420924	206	-0.01453230	-0.683	-0.00317190	-0.148
6	-.71885731	-.71873294	-.71892629	203	+.00006898	-.0096	-.00019335	-.0269
8	-.35995980	-.35995967	-.35997430	199	.00001450	-.0040	.00001463	-.0041
10	-.20220971	-.20220944	-.20221101	196	.00000130	-.0006	.00000157	-.0008
12	-.12837213	-.12837204	-.12837449	206	.00000236	-.0018	.00000245	-.0019
14	-.089105542	-.089105449	-.089106542	204	.00000100	-.0011	.00000109	-.0012
16	-.065896532	-.065896490	-.065897180	202	.00000065	-.0010	.00000069	-.0010
18	-.051034574	-.051034536	-.051034918	200	.00000034	-.0007	.00000038	-.0007
20	-.040918881	-.040918875	-.040918899	199	.00000002	-.0000	.00000002	-.0001
25	-.026177450	-.026177437	-.026177449	202	-.00000000	+.000000	+.00000001	-.0000
30	-.018508841	-.018508831	-.018509098	200	+.00000026	-.0014	+.00000027	-.0014
35	-.013960312	-.013960307	-.013960137	198	-.00000018	+.0013	-.00000017	+.0012
40	-.011017306	-.011017269	-.011017328	197	+.00000002	-.0002	+.00000000	-.0005
45	-.0089922416	-.0089922358	-.0089923444	196	+.00000010	-.0011	+.00000011	-.0012
50	-.0075336267	-.0075336268	-.0075335180	195	-.00000011	+.0014	-.00000011	+.0014
55	-.0064453260	-.0064453248	-.0064452224	197	-.00000010	+.0016	-.00000010	+.0016
60	-.0056103048	-.0056103108	-.0056103669	196	+.00000006	-.0011	+.00000006	-.0010
70	-.0044308652	-.0044308654	-.0044309059	192	+.00000004	-.0009	+.00000004	-.0009
80	-.0036549128	-.0036549233	-.0036549531	191	.00000004	-.0011	.00000003	-.0008
90	-.0031185937	-.0031185916	-.0031185585	191	-.00000004	+.0011	-.00000003	+.00011
100	-.0027346945	-.0027346779	-.0027346959	191	+.00000000	-.0001	+.00000002	-.0007
120	-.0022439739	-.0022440230	-.0022440170	196	+.00000004	-.0019	-.00000001	+.0008
140	-.0019721907	-.0019723686	-.0019721371	197	-.00000005	+.0027	-.00000023	+.0117
160	-.0018329593	-.0018328758	-.0018329125	199	-.00000005	+.0025	+.00000004	-.0020

TABLE IV. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION
 FOR SEVERAL VALUES OF RADIUS RATIO
 [Conical half-angle, 5° .]
 (b) $r/\infty = 0.96$

Field angle, φ , deg	Values of B_r obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
	Deviation	Percent deviation	Deviation	Percent deviation				
1	4.1953852	4.1953902	4.1954250	277	-0.00003980	-0.0009	-0.00003480	-0.0008
2	4.2967142	4.2967165	4.2967593	245	-0.00004510	-.0010	-0.00004280	-.0010
3	4.4865938	4.4865973	4.4866799	228	-0.00008610	-.0019	-0.00008260	-.0018
4	4.7875918	4.7875893	4.7877921	254	-0.00020030	-.0042	-0.00020280	-.0042
5	2.0216944	2.0330522	2.0362254	243	-.01453100	-.719	-.00317320	-.156
6	-.73980865	-.73968450	-.73987302	235	+.00006437	-.0087	-.00018852	-.0255
8	-.33361782	-.33361779	-.33363675	243	.00001893	-.0057	.00001896	-.0057
10	-.18166775	-.18166779	-.18166903	247	.00000128	-.0007	.00000124	-.0007
12	-.11495672	-.11495662	-.11495876	248	.00000204	-.0018	.00000214	-.0019
14	-.080214763	-.080214676	-.080215960	243	.00000120	-.0015	.00000128	-.0016
16	-.059770939	-.059770907	-.059770679	246	-.00000026	+.0004	-.00000023	+.0004
18	-.046649760	-.046649716	-.046650452	247	+.00000069	-.0015	.00000074	-.0016
20	-.037673537	-.037673526	-.037673806	243	.00000027	-.0007	.00000028	-.0007
25	-.024460415	-.024460394	-.024460300	244	-.00000012	+.0005	-.00000009	+.0004
30	-.017481888	-.017481876	-.017482131	241	+.00000024	-.0014	.00000026	-.0015
35	-.013289978	-.013289970	-.013290281	239	.00000030	-.0023	.00000031	-.0023
40	-.010550177	-.010550140	-.010550129	238	-.00000005	+.0005	-.00000001	+.0001
45	-.0086497515	-.0086497480	-.0086496030	236	-.00000015	+.0017	-.00000014	+.0017
50	-.0072721023	-.0072721013	-.0072722855	238	+.00000018	-.0025	+.00000018	-.0025
55	-.0062388683	-.0062388677	-.0062387340	240	-.00000013	+.0022	-.00000013	+.0021
60	-.0054427331	-.0054427391	-.0054426255	238	-.00000011	+.0020	-.00000011	+.0021
70	-.0043128157	-.0043128161	-.0043126945	239	-.00000012	+.28	-.00000012	+.0028
80	-.0035657097	-.0035657200	-.0035657564	239	+.00000005	-.0013	+.00000004	-.0010
90	-.0030474623	-.0030474593	-.0030475273	239	.00000006	-.0021	.00000007	-.0022
100	-.0026755071	-.0026754907	-.0026754235	239	-.00000008	+.0031	-.00000007	+.0025
120	-.0021987948	-.0021988432	-.0021987339	238	-.00000006	+.0028	-.00000011	+.0050
140	-.0019341357	-.0019343136	-.0019342272	238	+.00000009	-.0047	-.00000009	+.0045
160	-.0017983731	-.0017982896	-.0017982961	243	-.00000008	+.0043	+.00000001	-.0004

TABLE IV. - Continued. VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION

FOR SEVERAL VALUES OF RADIUS RATIO

[Conical half-angle, 5° .](c) $r/R = 0.97$

Field angle, ϕ , deg	Values of B_r obtained by method of -			Number of terms	Values obtained by method of -			
	Gauss	Simpson	Legendre		Gauss		Simpson	
	Deviation	Percent deviation	Deviation	Percent deviation				
1	3.8816945	3.8816995	3.8817270	301	-0.00003250	-0.0008	-0.00002750	-0.0007
2	3.9788415	3.9788444	3.9788693	301	-.00002780	-.0007	-.00002490	-.0006
3	4.1742145	4.1742175	4.1742780	287	-.00006350	-.0015	-.00006050	-.0014
4	4.5273446	4.5273440	4.5275401	301	-.00019550	-.0043	-.00019610	-.0043
5	1.9012901	1.9126481	1.9158224		-.01453230	-.764	-.00317430	-.166
6	-.73337753	-.73325132	-.73343993		+.00006240	-.0085	.00018861	-.0257
8	-.29290110	-.29290070	-.29291343		.00001233	-.0042	.00001273	-.0043
10	-.15692791	-.15692785	-.15693241		.00000450	-.0029	.00000456	-.0029
12	-.10015456	-.10015440	-.10015579		.00000123	-.0012	.00000139	-.0014
14	-.070807043	-.070806938	-.070806547		-.00000050	+.0007	-.00000039	+.0006
16	-.053439597	-.053439545	-.053440543		+.00000095	-.0018	+.00000100	-.0019
18	-.042183303	-.042183262	-.042183458		.00000016	-.0004	.00000020	-.0005
20	-.034399787	-.034399788	-.034399492		-.00000030	+.0009	-.00000030	+.0009
25	-.022752207	-.022752193	-.022751991		-.00000022	+.0009	-.00000020	+.0009
30	-.016467354	-.016467346	-.016467377		+.00000002	-.0001	.00000003	-.0002
35	-.012630422	-.012630416	-.012630581		.00000016	-.0013	.00000016	-.0013
40	-.010091733	-.010091697	-.010091839		.00000011	-.0011	.00000014	-.0014
45	-.0083142046	-.0083141998	-.0083141848		-.00000002	+.0002	-.00000002	+.0002
50	-.0070161846	-.0070161853	-.0070160993		-.00000009	+.0012	-.00000009	+.0012
55	-.0060370122	-.0060370112	-.0060369669		-.00000005	+.0008	-.00000004	+.0007
60	-.0052790031	-.0052790089	-.0052790378		+.00000003	-.0007	+.00000003	-.0005
70	-.0041975731	-.0041975736	-.0041975940		.00000002	-.0005	.00000002	-.0005
80	-.0034786753	-.0034786849	-.0034786257		-.00000005	+.0014	-.00000006	+.0017
90	-.0029780835	-.0029780810	-.0029781232		+.00000004	-.0013	+.00000004	-.0014
100	-.0026177934	-.0026177764	-.0026177889		-.00000000	+.0002	+.00000001	-.0005
120	-.0021547521	-.0021548013	-.0021547972		+.00000005	-.0021	-.00000000	+.0002
140	-.0018970435	-.0018972218	-.0018970245		-.00000002	+.0010	-.00000020	+.0104
160	-.0017646623	-.001245824	-.0017646602		-.00000000	+.0001	+.00000008	-.0044

TABLE IV. - Concluded. VALUES OF MAGNETIC FIELD COMPONENT IN r -DIRECTION

FOR SEVERAL VALUES OF RADIUS RATIO

[Conical half-angle, 5° .](d) $r/\alpha = 0.99$

Field angle, φ , deg	Values of B_r , Gauss	obtained by method of -			Number of terms	Values obtained by method of -			
		Simpson	Legendre	Gauss		Simpson			
				Deviation		Percent deviation	Deviation	Percent deviation	
1	3.1715743	3.1715811	3.1692656	301	0.00230870	0.0728	0.00231550	0.730	
2	3.2215559	3.2215601	3.2149937		.00656220	.2037	.00656640	.2038	
3	3.3338127	3.3338187	3.3283256		.00548710	.1646	.00549310	.1648	
4	3.6235383	3.6235394	3.6229809		.00055740	.0154	.00055850	.0154	
5	1.5935673	-----	1.6090732		-.01550590	-.9730	-----	-----	
6	-.48456996	-.48444638	-.48311913		-.00145083	.2994	-.00132725	.2740	
8	-.16782217	-.16782185	-.16594698		-.00187519	1.1174	-.00187487	1.1172	
10	-.098099660	-.098095342	-.098941176		+.00084152	-.8578	.00084583	-.8623	
12	-.067944602	-.067944477	-.068018757		+.00007416	-.1091	.00007428	-.1093	
14	-.051171350	-.051171242	-.050765056		-.00040629	+.7940	+.00040619	+.7938	
16	-.040527733	-.040527690	-.040800189		+.00027246	-.6723	.00027250	-.6724	
18	-.033204370	-.033204320	-.033221686		+.00001732	-.0521	.00001739	-.0524	
20	-.027881133	-.027874047	-.027696519		-.00018461	+.6621	-.00017753	+.6369	
25	-.019396748	-.019396730	-.019285454		-.00011129	.5738	-.00011128	.5737	
30	-.014488122	-.014488112	-.014496269		+.00000815	-.0562	+.00000816	-.0563	
35	-.011348803	-.011348796	-.011415414		.00006661	-.5869	.00006662	-.5870	
40	-.0092031232	-.0092030894	-.0092461307		.00004301	-.4673	.00004304	-.4677	
45	-.0076648926	-.0076648884	-.0076522014		-.00001269	+.1656	-.00001269	+.1655	
50	-.0065215451	-.0065215446	-.0064821092		-.00003944	+.6047	-.00003944	+.6047	
55	-.0056471965	-.0056471937	-.0056276476		-.00001955	+.3462	-.00001955	+.3461	
60	-.0049630139	-.0049630196	-.049792581		+.00001624	-.3273	.00001624	-.3272	
70	-.0039753546	-.0039753549	-.0039835721		+.00000822	-.2067	.00000822	-.2067	
80	-.0033109379	-.0038109491	-.0032886001		-.00002234	+.6747	-.00002235	+.6750	
90	-.0028444206	-.0028444186	-.0028636461		+.00001923	-.6759	.00001923	-.6760	
100	-.0025066290	-.0025066120	-.0025046937		-.00000194	+.0772	-.00000192	+.0765	
120	-.0020699451	-.002069936	-.0020889546		+.00001901	-.9184	.00001896	-.9160	
140	-.0018256293	-.0018258080	-.0018153202		-.00001031	+.5647	-.00001049	+.5744	
160	-.0016997673	-.0016996865	-.0016871792		-.00001259	+.7406	-.00001251	+.7359	

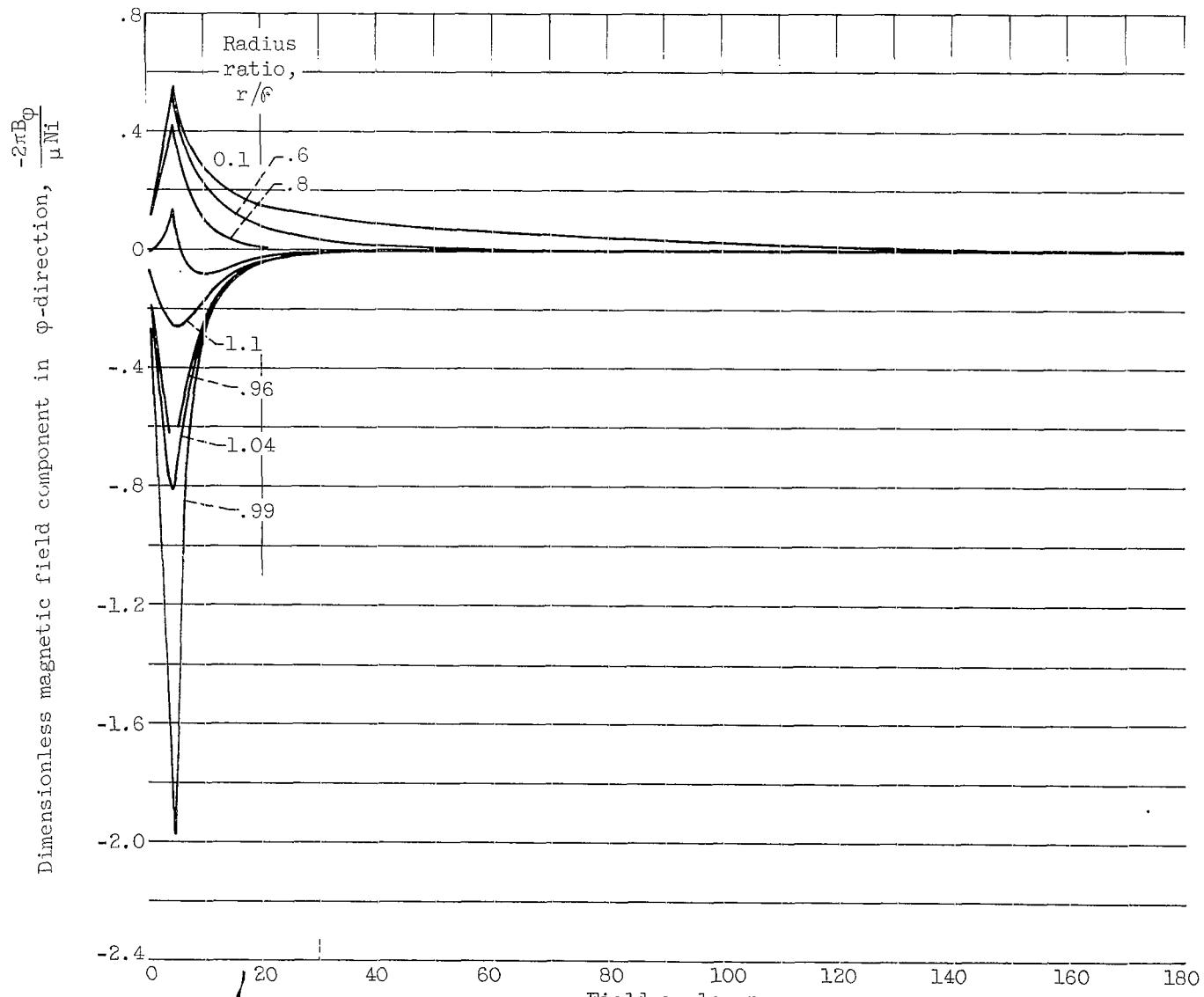
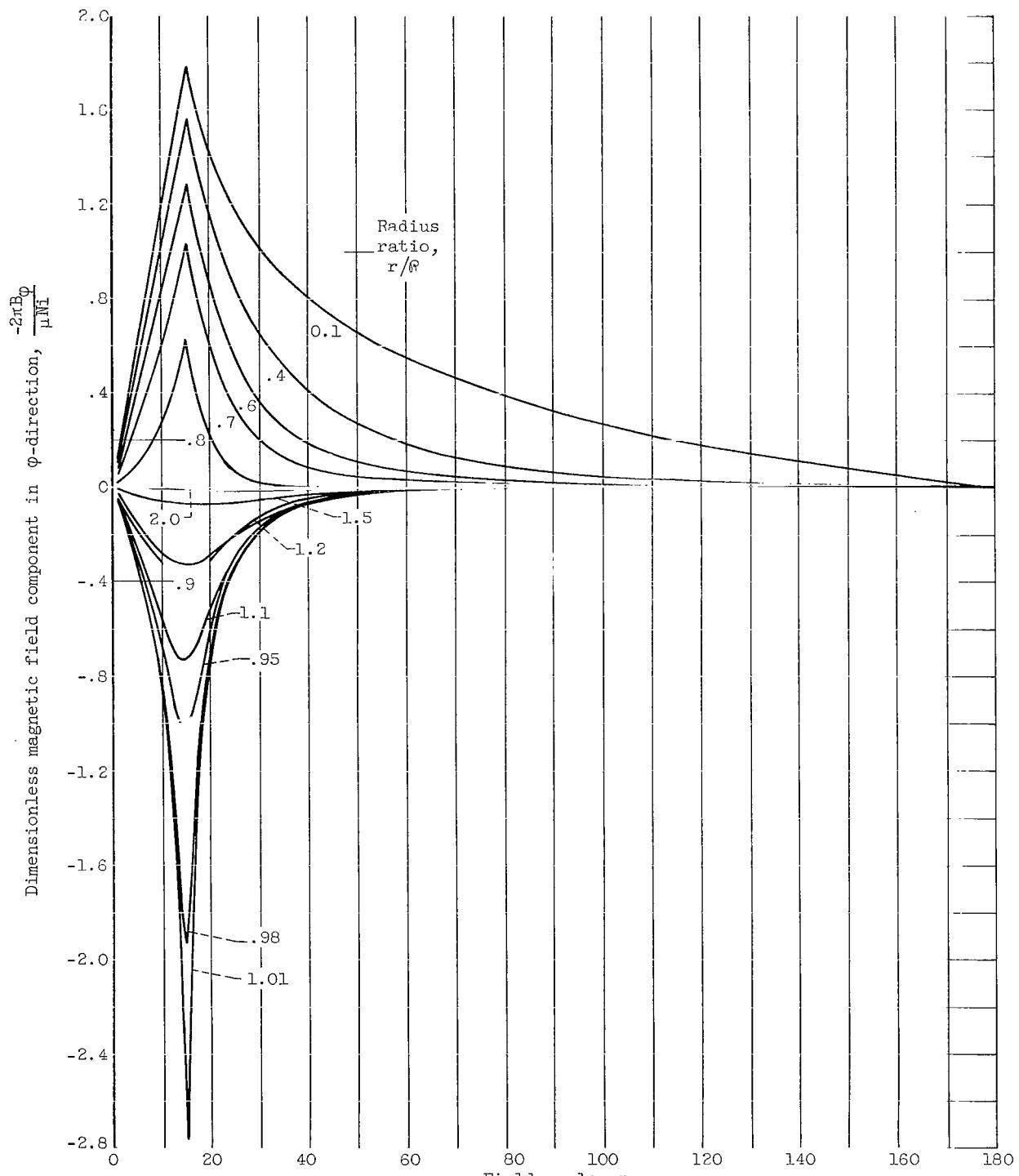
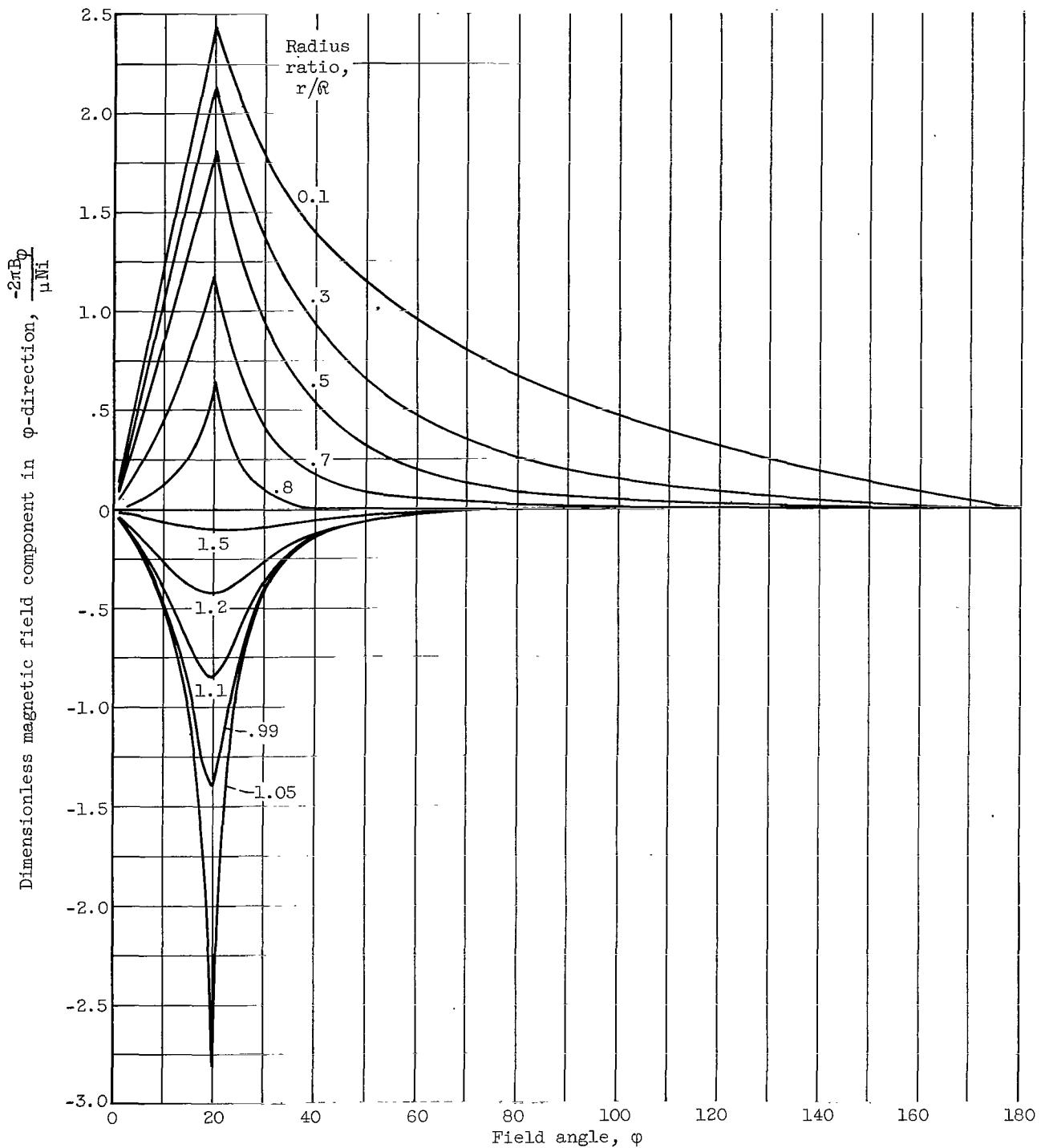
(a) Conical half-angle, 5° .

Figure 1. - Dimensionless magnetic field component as function of field angle for various values of radius ratio.



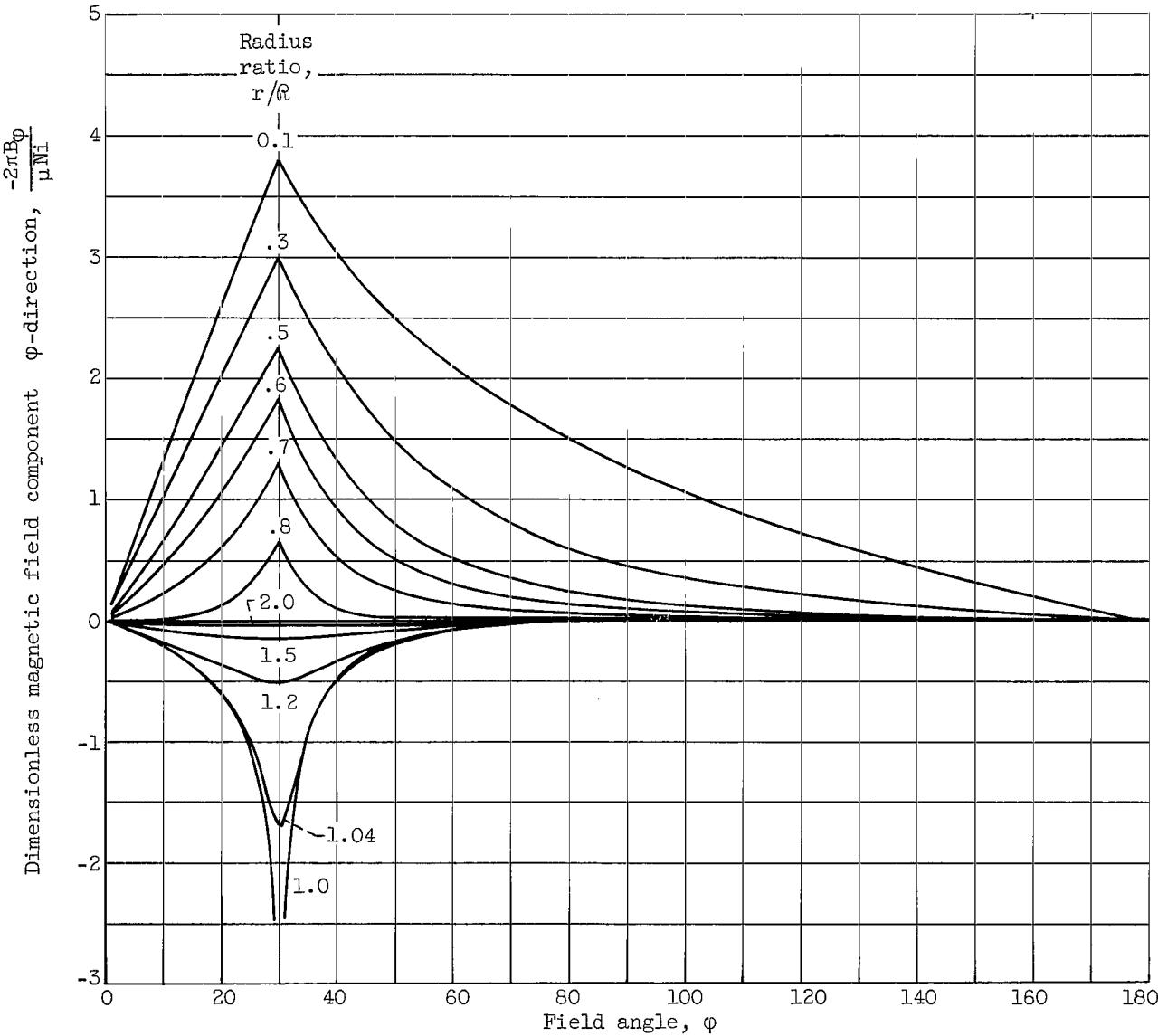
(b) Conical half-angle, 15° .

Figure 1. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



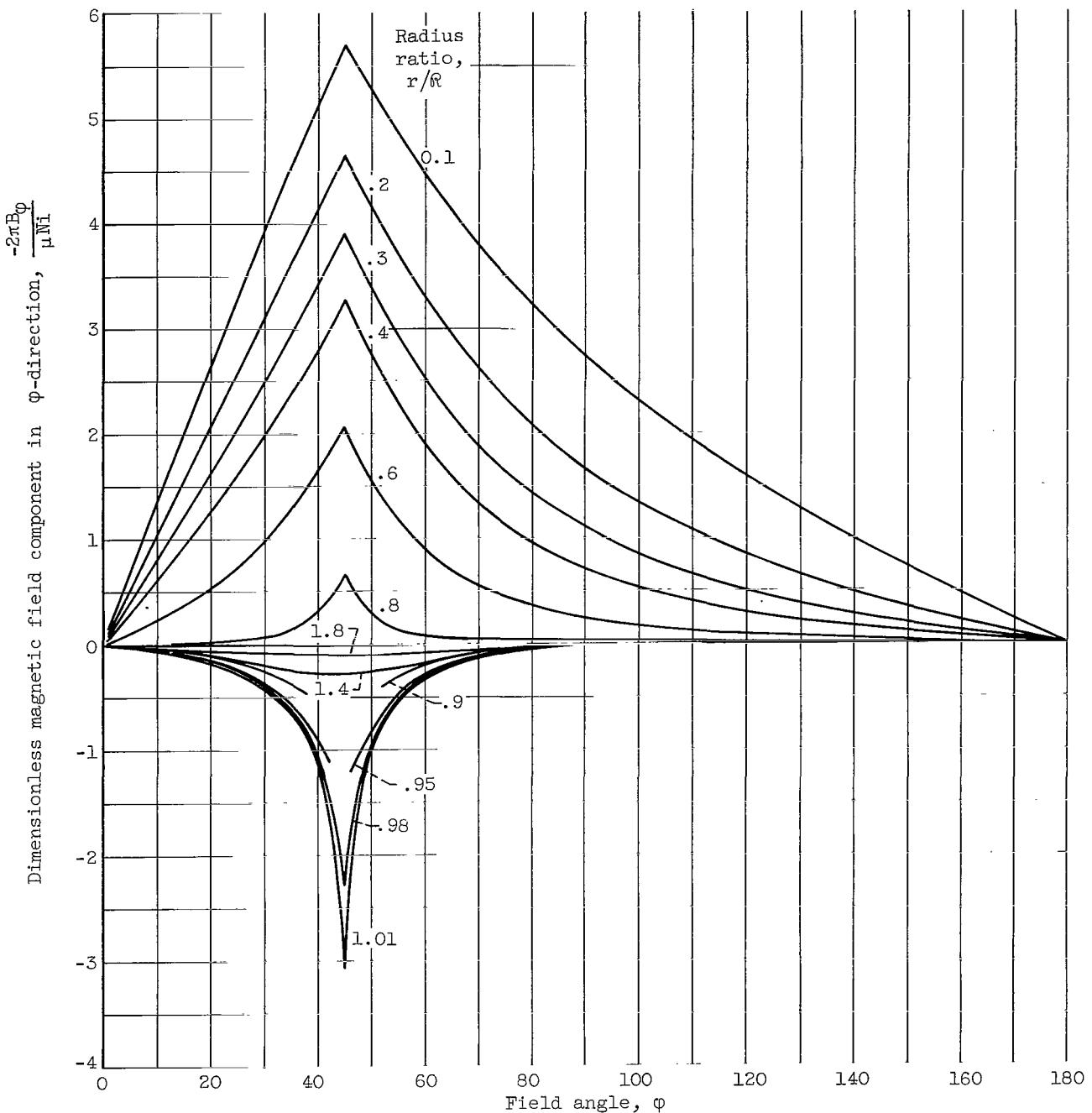
(c) Conical half-angle, 20° .

Figure 1. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



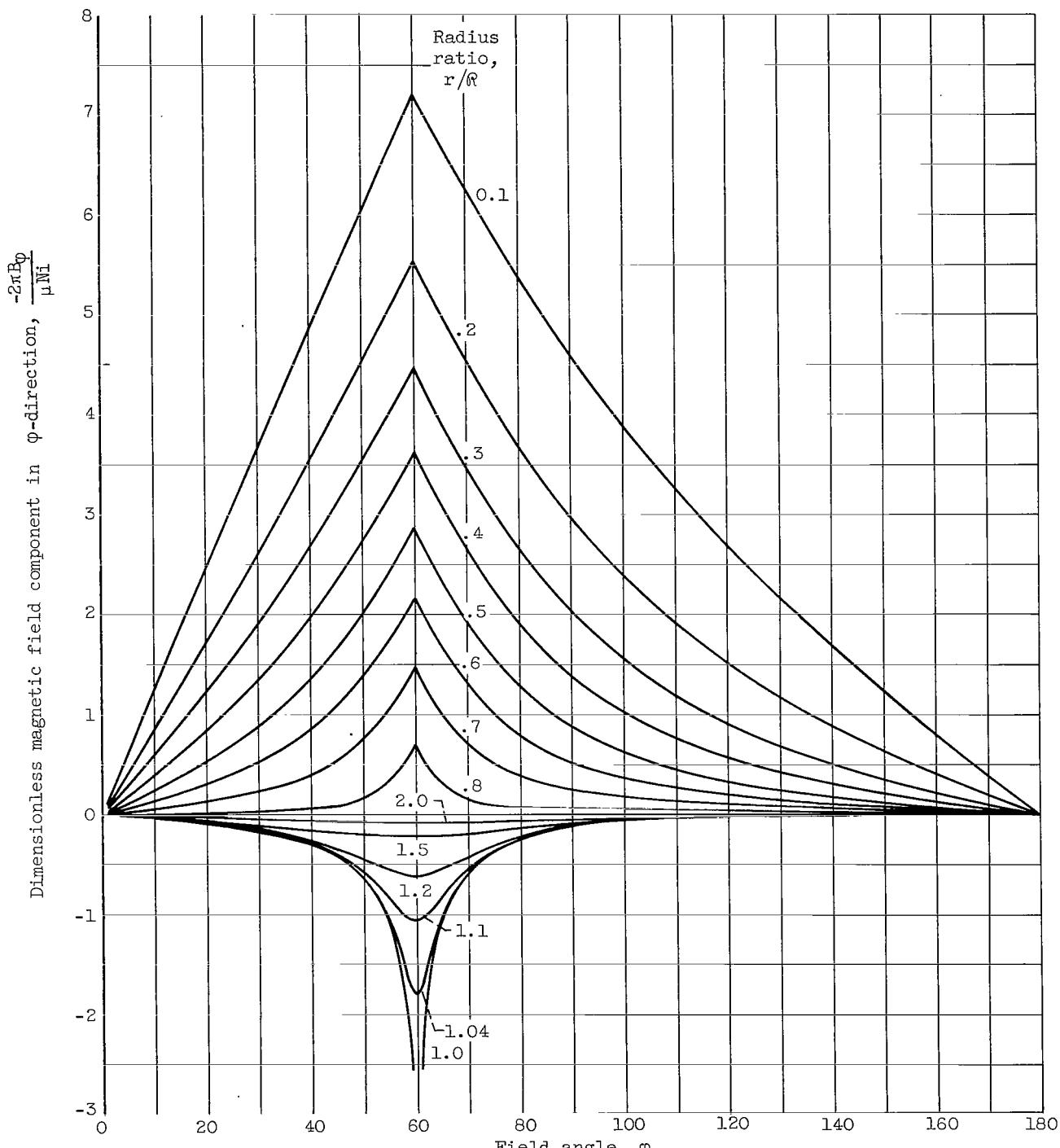
(d) Conical half-angle, 30° .

Figure 1. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



(e) Conical half-angle, 45° .

Figure 1. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



(f) Conical half-angle, 60° .

Figure 1. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.

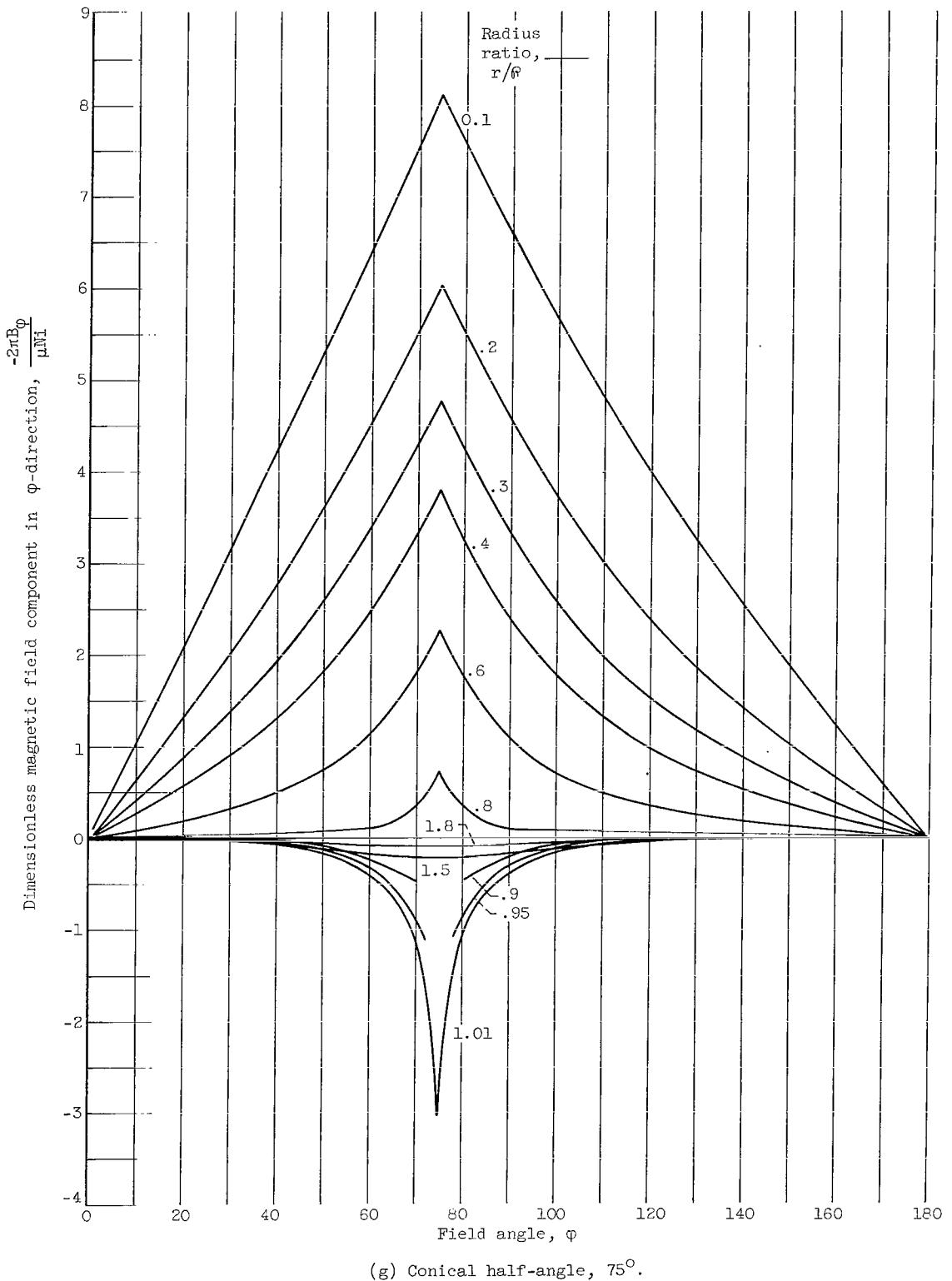
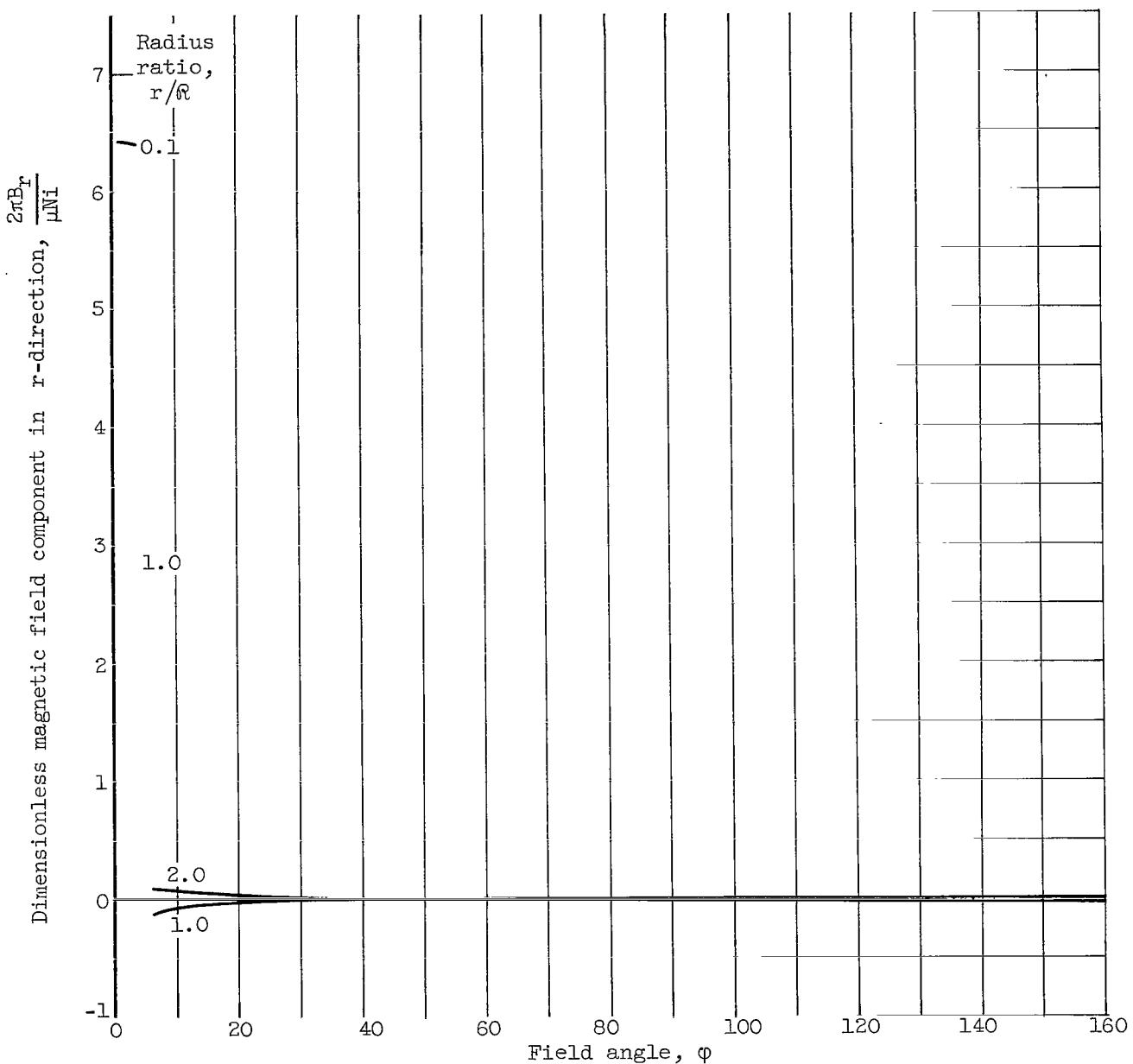
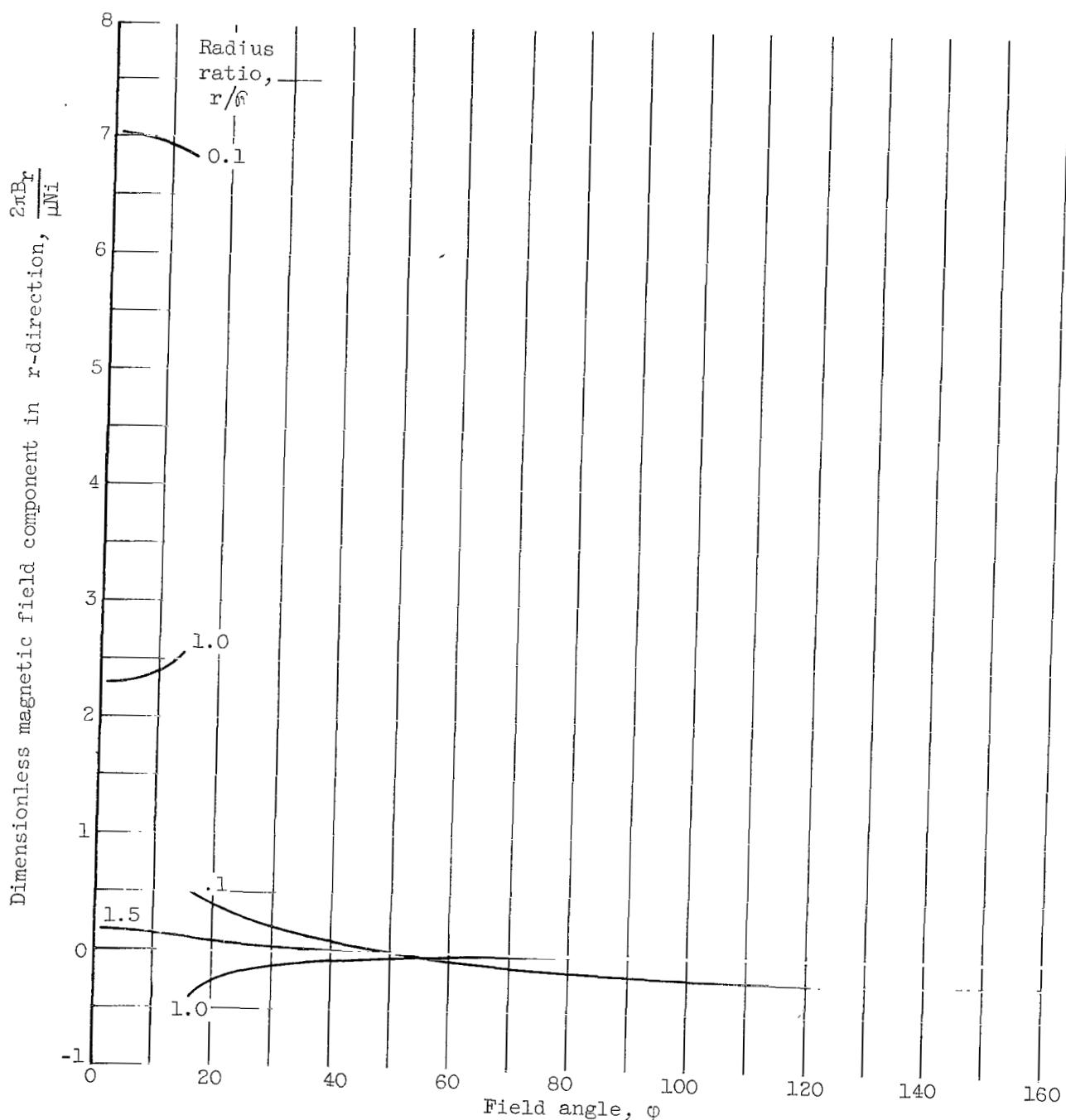


Figure 1. - Concluded. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



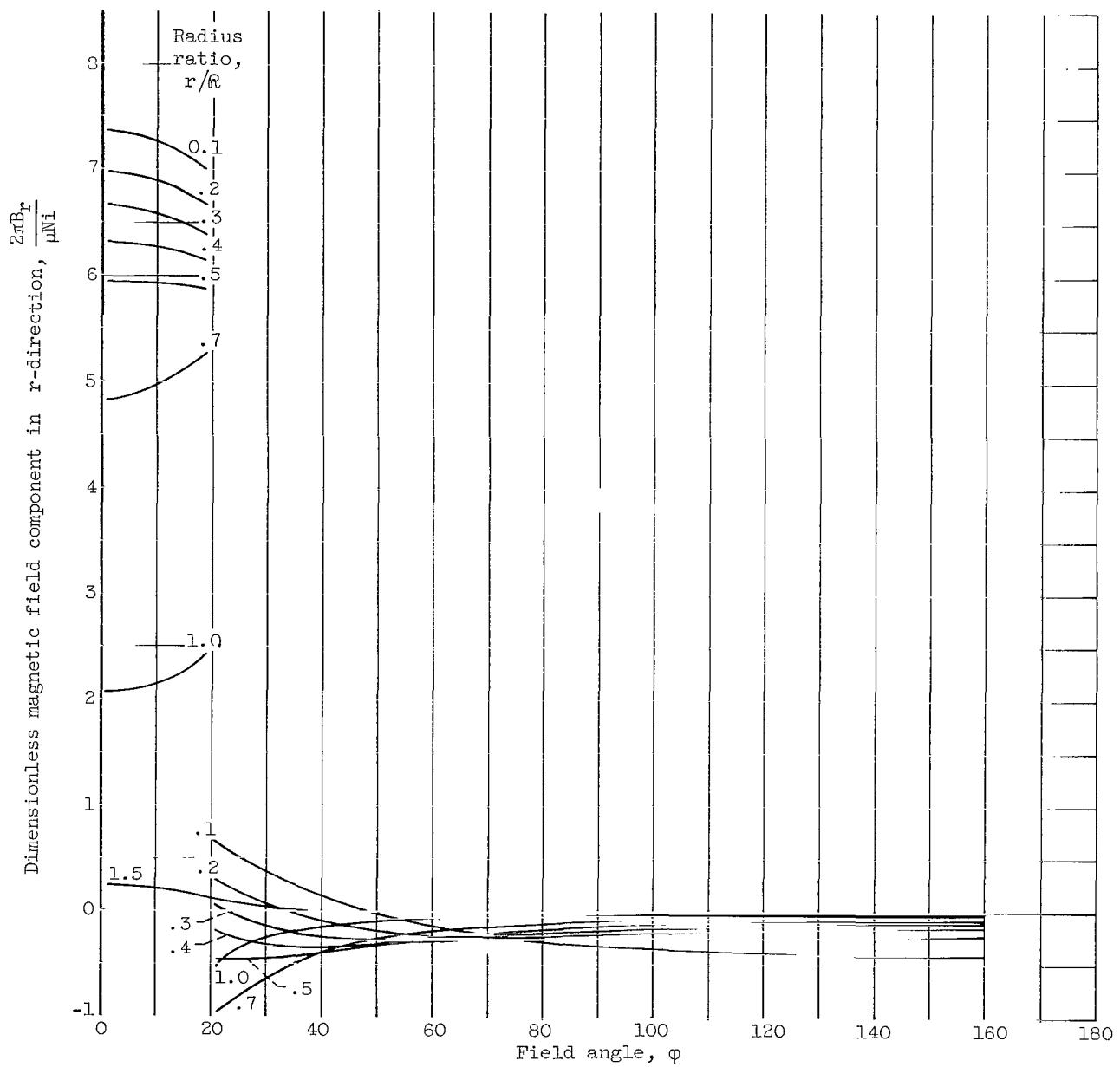
(a) Conical half-angle, 5° .

Figure 2. - Dimensionless magnetic field component as function of field angle for various values of radius ratio.



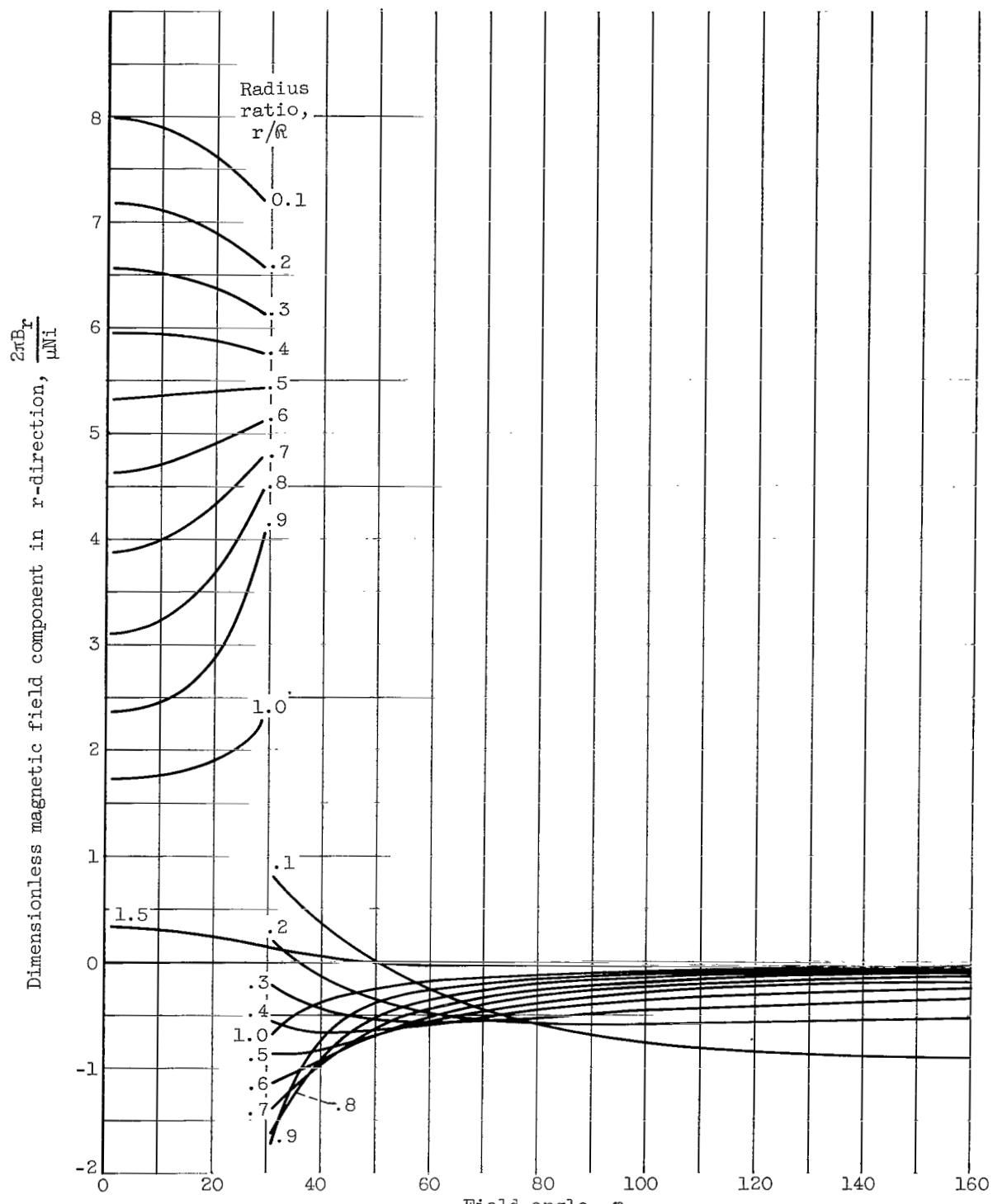
(b) Conical half-angle, 15° .

Figure 2. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



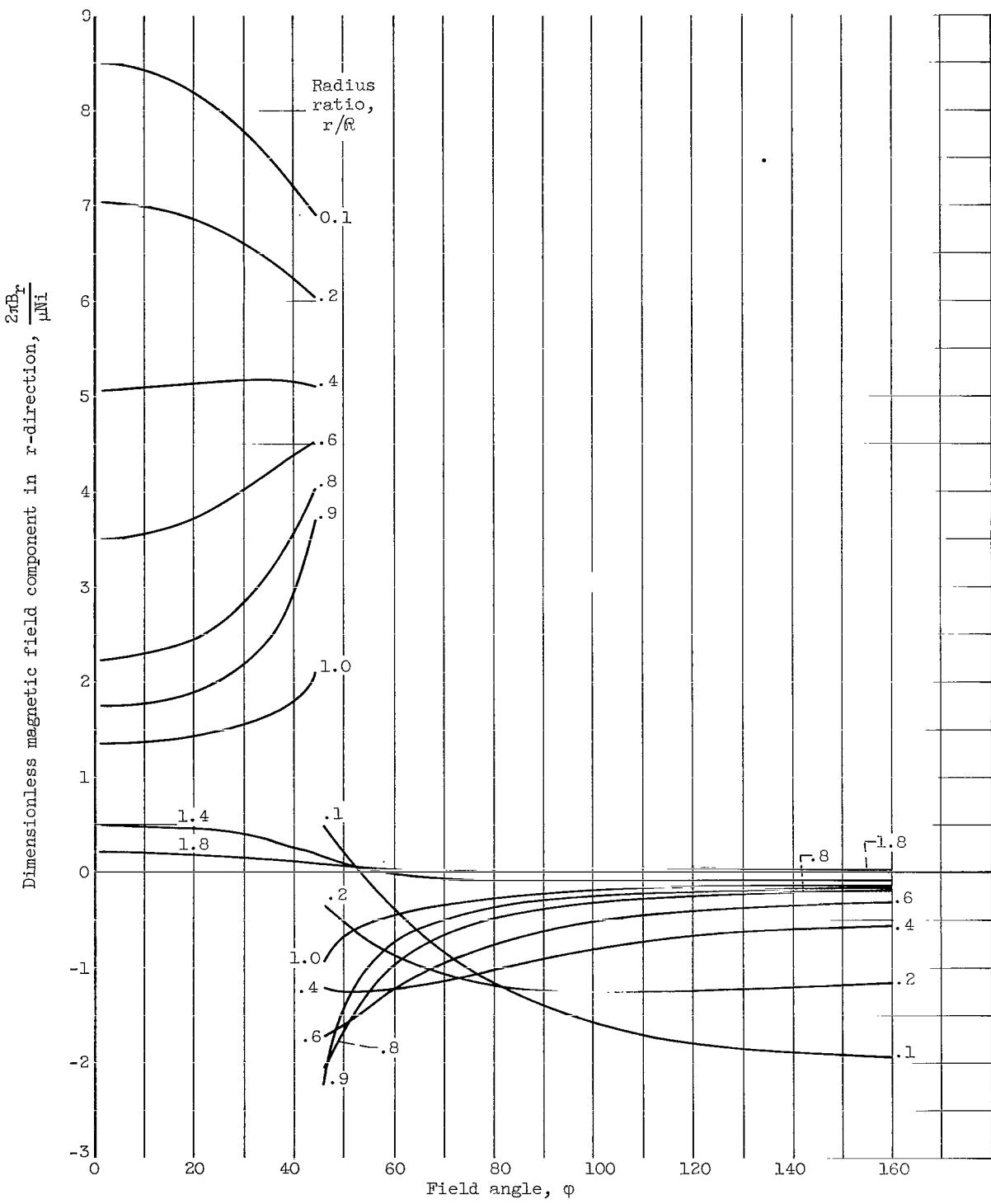
(c) Conical half-angle, 20° .

Figure 2. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



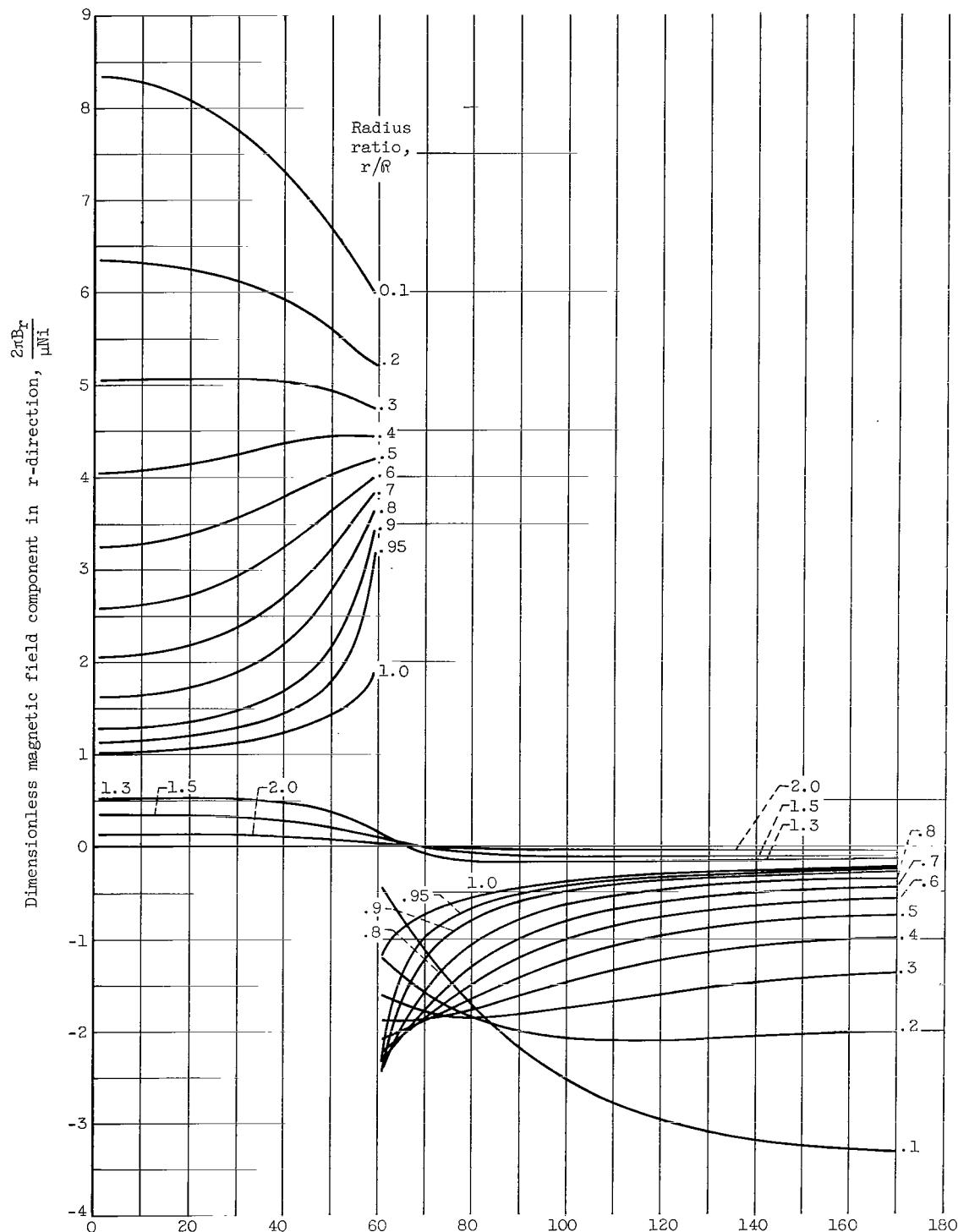
(d) Conical half-angle, 30° .

Figure 2. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



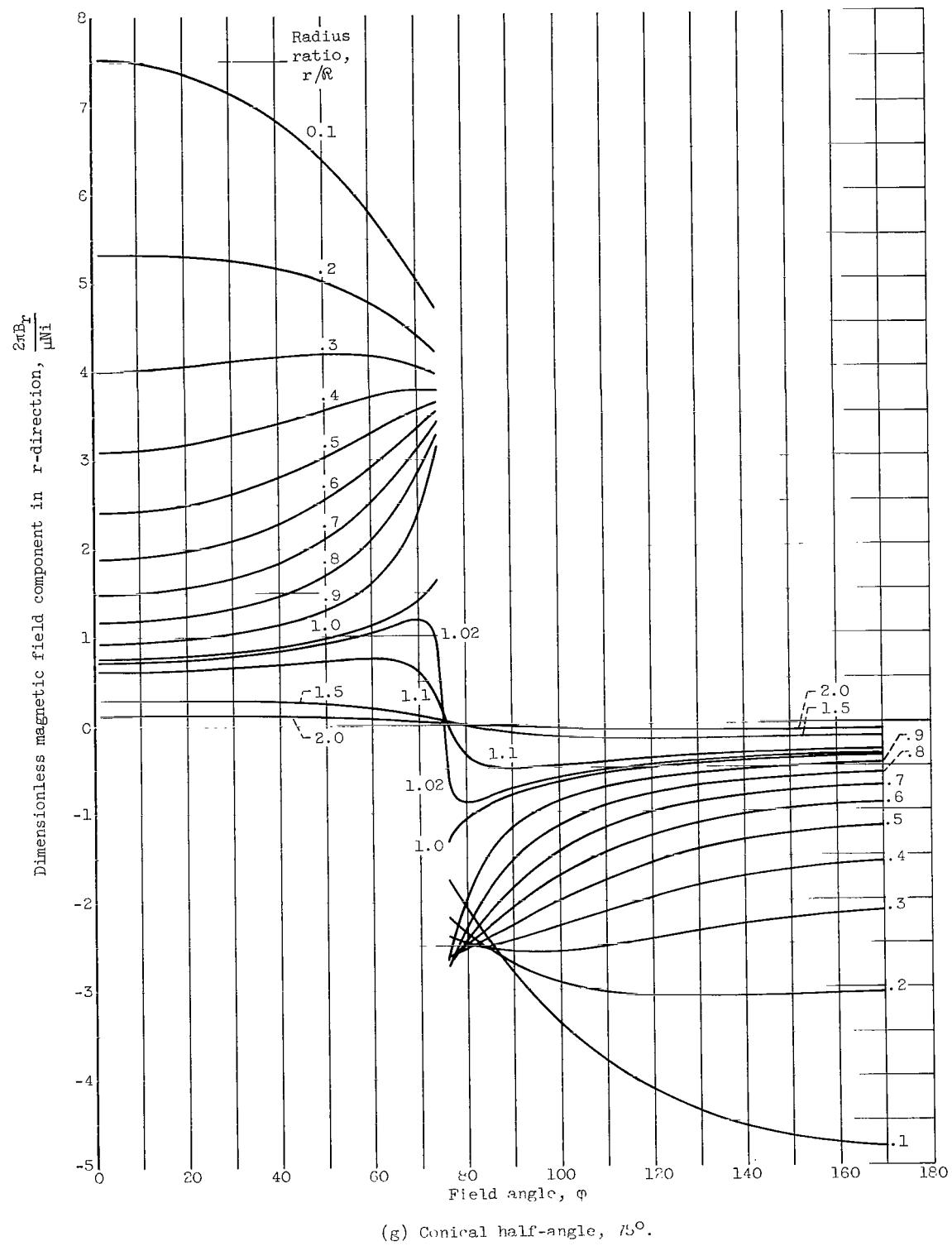
(e) Conical half-angle, 45° .

Figure 2. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



(f) Conical half-angle, 60° .

Figure 2. - Continued. Dimensionless magnetic field component as function of field angle for various values of radius ratio.



(g) Conical half-angle, 75° .

Figure 2. - Concluded. Dimensionless magnetic field component as function of field angle for various values of radius ratio.

2/7 (D)
J

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